ABSTRACTS FOR THE WORKSHOP IN ANALYSIS, ATLANTA, GA, DECEMBER 8-10, 2023.

Catherine Beneteau, University of South Florida

Title: Minimal Zeros, orthogonal polynomials, and Jacobi matrices revisited

Abstract: In this talk, I will revisit some work that appeared in 2016 (Beneteau, Khavinson, Liaw, Seco, and Simanek) that studied minimal zeros of optimal polynomial approximants in certain Hilbert spaces of analytic functions, and the connection between this problem, orthogonal polynomials on the real line, Jacobi matrices, and some particular differential equations. The goal of the talk is to discuss these connections and some interesting remaining open problems.

Peter Binev, University of South Carolina

Title: Solving PDEs with Incomplete Information

Abstract: We consider the problem of numerically approximating the solutions to a PDE when there is insufficient information to determine a unique solution. Our main example is the Poisson boundary value problem, when the boundary data is unknown and instead one observes finitely many linear measurements of the solution described by some linear functionals. We view this setting as an optimal recovery problem and develop theory and numerical algorithms for its solution. The main vehicle employed is the derivation and approximation of the Riesz representers of these functionals with respect to relevant Hilbert spaces of harmonic functions. This is a joint research with Andrea Bonito, Albert Cohen, Wolfgang Dahmen, Ronald DeVore, and Guergana Petrova. A preprint is available at arXiv:2301.05540 [math.NA].

Adam Black, Yale University

Title: Dispersion for Coulomb waves

Abstract: We study the Schrödinger equation with a repulsive Coulombic potential on $\mathbb{R}^3$. For radial data, we obtain an $L^1 -> L^\infty$ dispersive estimate with the natural decay rate $t^{-\frac{3}{2}}$. Our proof uses the spectral theory of strongly singular potentials to obtain an expression for the evolution kernel. A semiclassical turning point analysis of the kernel then allows the time decay to be extracted via oscillatory integral estimates. This is joint work with E. Toprak, B. Vergara, and J. Zou.

Alan Chang, Washington University in St. Louis

Title: Embedding snowflakes of the Heisenberg group into Euclidean space

Abstract: One consequence of Assouad's embedding theorem is that the snowflaked Heisenberg group has a bi-Lipschitz embedding into Euclidean space. (Those terms will be defined in the talk.) Terence Tao improved this result by constructing an embedding which is in some sense optimal. His proof uses the Nash-Moser iteration

scheme, Littlewood-Paley theory on the Heisenberg group, and quantitative homotopy lifting arguments. (Those terms will not be defined in the talk.) We present an alternative proof of Tao's result which relies primarily on the lattice structure of the Heisenberg group as well as one of the oldest tricks in harmonic analysis. This is joint work with Seung-Yeon Ryoo.

Manuel Fernandez, Georgia Tech

Title: On the ℓ_0-Isoperimetry of Measurable Sets

Abstract: Gibbs-sampling, also known as coordinate hit-and-run (CHAR), is a random walk used to sample points uniformly from convex bodies. Its transition rule is simple: Given the current point p, pick a random coordinate i and resample the i'th coordinate of p according to the distribution induced by fixing all other coordinates. Despite its use in practice, strong theoretical guarantees regarding the mixing time of CHAR for sampling from convex bodies were only recently shown in works of Laddha and Vempala, Narayanan and Srivastava, and Narayanam, Rajaraman and Srivastava. In the work of Laddha and Vempala, as part of their proof strategy, the authors introduced the notion of the ℓ_0 isoperimetric coefficient of a measurable set and provided a lower bound for the quantity in the case of axis-aligned cubes. In this talk we will present some new results regarding the ℓ_0 isoperimetric coefficient of measurable sets. In particular we pin down the exact order of magnitude of the ℓ_0 isoperimetric coefficient of axis-aligned cubes and present a general upper bound of the ℓ_0 isoperimetric coefficient for any measurable set. As an application, we will mention how the results give a moderate improvement in the mixing time of CHAR.

Christina Giannitsi, Vanderbilt

Title: Gaussian Integers: Prime averages and the Goldbach Conjecture.

Abstract: Inspired by previous work on averaging operators in the discrete setting and their number-theoretic connections, we study averages taken over the Gaussian primes and prove ℓ^p-improving and weighted inequalities. We than apply our estimates to obtain density results for the binary and ternary version of Goldbach's Conjecture. This is joint work with Ben Krause, Michael Lacey, Hamed Mousavi and Yaghoub Rahimi.

Rachel Greenfeld, IAS

Title: Tiling, spectrality and aperiodicity of connected sets

Abstract: Let $\Omega \subset \mathbb{R}^d$ be a set of finite measure. The periodic tiling conjecture asserts that if Ω tiles $\mathbb{R}^d$ by translations then it admits at least one periodic tiling. Fuglede's conjecture suggests that Ω admits an orthogonal basis of exponential functions if and only if it tiles $\mathbb{R}^d$ by translations. Both conjectures are known to be false in sufficiently high dimensions, with all the so-far-known counterexamples being disconnected. On the other hand, both conjectures hold for convex sets. In the talk I will survey the study of these conjectures, and discuss a joint work with Mihalis Kolountzakis where we construct *connected* counterexamples to the periodic tiling conjecture as well as to both directions of Fuglede's conjecture.

Chris Heil, Georgia Tech

Title: Overcomplete Reproducing Pairs

Abstract: The Gaussian Gabor system at the critical density has the property that it is overcomplete in $L^2(\mathbb{R})$ by exactly one element, and if any single element is removed then the resulting system is complete but is not a Schauder basis. In this talk we will characterize systems that are overcomplete by finitely many elements. Among other results, we will show that if such a system has a reproducing partner, then it contains a Schauder basis. While a Schauder basis provides a strong reproducing property for elements of a space, the existence of a reproducing partner only requires a weak type of representation of elements. Thus for these systems weak representations imply strong representations. The results are applied to systems of weighted exponentials and to Gabor systems at the critical density. In particular, we show that the Gaussian Gabor system does not possess a reproducing partner. This work is joint with Logan Hart, Ian Katz, and Michael Northington.

Orli Herscovici, St. John's University, NY

Title: On stability and equality in the B-theorem

Abstract: In this talk we present our recent work investigating stability in the B-inequaity of Cordero-Erausquin, Fradelizi, and Maurey. Moreover, we show that equality holds when the symmetric convex set is either has an empty interior or is the whole space. This talk is based on the joint work with Galyna Livshyts, Liran Rotem, and Alexander Volberg.

John Hoffman, Florida State University

Title: Big pieces approximation by Lipschitz graphs in the parabolic setting.

Abstract: I will present a result about big pieces approximation by Lipschitz graphs in the parabolic setting. This is a parabolic analog of a result of David and Jerison in the standard Euclidean setting. I will spend time reviewing notions of uniform rectifiability, and then present the ideas in proof of our result.

Plamen Iliev, Georgia Tech

Title: Bernstein-Szego measures in the plane

Abstract: I will define a class of Bernstein-Szegő measures on $\mathbb{R}^2$ which provides a natural extension of the one-dimensional theory. I will discuss their spectral properties and conditions in the two-dimensional setting involving finitely many moments which completely characterize them. The talk is based on joint work with J. Geronimo.

Alex Iosevich, University of Rochester

Title: Exact signal recovery and restriction theory

Abstract: We are going to discuss a classical problem, studied by Donoho and Stark, and many others, where a signal is encoded using the finite Fourier transform and a subset of the Fourier coefficients are lost. The question is, how many coefficients

can we lose and still recover the signal exactly. A variety of intriguing connections with discrete and continuous restriction theory arise naturally.

Dmitry Khavinson, University of South Florida

Title: Approximation Theory and Some Free Boundary Problems

Abstract: I shall try to outline the program started 4 decades ago that focuses on approximating particular “badly approximable” functions that leads to unexpectedly exciting problems in mathematical physics. So called free boundary problems. For example studying the approximation of z^* by analytic functions leads to the isoperimetric inequality, J. Serrin's problem in hydrodynamics, equilibrium shape of electrified droplets, St. Venant's inequality for torsional rigidity, etc. Many simply stated open questions, that still remain open in spite of the recent progress in the last decades, will be mentioned. The talk should be accessible to everyone

Lyudmila Kryvonos, Vanderbilt University

Title: A constrained logarithmic energy problem on the unit circle

Abstract: We study the problem of minimizing the logarithmic energy, $\mathcal{E}(\mu) := \iint \log \frac{1}{|z-\zeta|} d\mu(z) d\mu(\zeta)$, of probability measures μ supported on the unit circle with an additional constraint imposed on the mass of a fixed subarc. Namely, for given θ, $0 < \theta < 2\pi$, and given q, $0 < q < 1$, we determine the measure ν, such that $\mathcal{E}(\neq) = \inf\{\mathcal{E}(\mu) : \mu \in \mathcal{P}(\mathbb{S}^1), \mu(A_\theta) = q\}$, where A_θ is the arc from $e^{-i\theta/2}$ to $e^{i\theta/2}$. The result answers a question raised by E. Meckes in connection with the characterization of behavior of eigenvalues of the kernel of the unitary eigenvalue process.

Dylan Langharst, Sorbonne

Title: Weighted Minkowski's Existence Theorem

Abstract: Minkowski once considered: what are the necessary and sufficient conditions for a collection of unit vectors $\{u_i\}$ and positive numbers $\{a_i\}$ to correspond to the convex polytope whose ith face has outer-unit normal u_i and surface area a_i? His famed existence theorem found there are merely two minor conditions needed to guarantee the existence of a unique, centered polytope with the prescribed properties. For an arbitrary convex body, whose resolution follows from the polytope case, one shows a Borel measure ν on the unit sphere is the surface area measure (S_K) for a unique centered convex body K.

In this talk, we consider replacing volume with some Borel measure that has density. We consider a rich class of Borel measures and solve the following weighted Minkowski's existence theorem: for ν a finite, even Borel measure on the unit sphere and μ an even Borel measure on $\mathbb{R}^n$ from the rich class, there exists a symmetric convex body K in $\mathbb{R}^n$ such that

$$d\nu(u) = c_{\mu,K} dS_{\mu,K}(u),$$

where $c_{\mu,K}$ is a quantity that depends on μ and K and $dS_{\mu,K}(u)$ is the surface area-measure of K with respect to μ. Examples of measures in the rich class are homogeneous measures (with $c_{\mu,K} = 1$) and radially decreasing probability measures with

continuous densities (e.g. the Gaussian measure). Under certain concavity conditions on the measure, we also obtain uniqueness.

Joint with L. Kryvonos.

Doron Lubinsky, Georgia Tech

Title: Erdos-Szekeres Polynomials

Abstract: In a 1959 paper, Erdos and Szekeres posed a number of problems on what are now called Erdos-Szekeres polynomials. They are polynomials of one variable with all zeros at roots of unity. The main problems are still unsolved. There are connections to several other topics, including combinatorics, number theory, the Prouhet-Tarry-Escott problem, and mathematical physics. We survey this at an introductory level.

Neil Lyall, University of Georgia

Title: Graham's Conjecture in Geometric Ramsey Theory

Abstract: During the 1970's Erdos, Graham, Montgomery, Rothschild, Spencer, and Straus initiated a study of geometric point configurations which cannot be destroyed by finite partitions of high dimensional Euclidean spaces. They showed that such configurations must be spherical and in 1994 Graham conjectured that this condition is also sufficient. While these problems were initially studied by purely combinatorial means a surprising density analogue has been shown for simplices by Bourgain, using Fourier analysis. We will present some recent results toward this conjecture both in the Euclidean setting and in vector spaces over finite fields, and discuss an approach utilizing some modern tools of additive combinatorics, also referred to as "higher order Fourier analysis".

Jose Madrid, Virginia Tech

Title: Analysis on the hypercube

Abstract: The set of vertices of a unit cube is called the hypercube. It consists of vectors with coordinates zeros and ones. In this talk we will discuss some series of analytic inequalities for subsets of the hypercube, lying at the interface of analysis, combinatorics and probability. We will focus on some recently obtained isoperimetric inequalities.

Akos Magyar, University of Georgia

Title: On a conjecture of Bergelson-Leibman in ergodic theory

Abstract: The classical Weyl equidistribution theorem may be viewed as an extension of von-Neumann's L^2-ergodic theorem to polynomial orbits in measure preserving systems. The corresponding pointwise polynomial ergodic theorem was established by Bourgain while the mean ergodic theorem was extended the nilpotent group actions by Bergelson and Leibman. These results led the far-reaching conjecture of Bergelson-Leibman on linear and multilinear polynomial orbits of nilpotent group actions.

We discuss various recent results toward this conjecture as well as our joint work on extending Bourgain's ergodic theorem to nilpotent group actions, based on developing an extension of the classical circle method of exponential sums to the nilpotent setting.

Michelle Mastrianni, University of Minnesota

Title: Fixed radius spherical cap discrepancy

Abstract: A seminal result of Beck shows that for any set of N points on the d-dimensional sphere, there always exists a spherical cap such that the number of points in the cap deviates from the expected value by at leas $N^{1/2-1/(2d))}$.

We refine the result by removing a layer of averaging: we show that, when d is not 1 mod 4, there exists a set of real numbers such that for each $r > 0$ in the set one is always guaranteed to find a spherical cap with radius r for which Beck's result holds. The main ingredient is a generalization of the notion of badly approximable numbers to the setting of Gegenbauer polynomials, which we call Gegenbadly approximable numbers. These are fixed numbers x such that the sequence of Gegenbauer polynomials evaluated at x avoids being close to 0 in a precise quantitative sense.

Azita Mayeli, SUNY

Title: Quantitative bounds on the distribution of eigenvalues for the spatio-spectral limiting operators

Abstract: The spatio-spectral limiting operator is the natural analog of the time-frequency limiting operator in one dimension extended to higher dimensions. In one dimension, the eigenfunctions of the time-frequency limiting operator are the prolate spheroidal wave functions. These functions are bandlimited, maximizing the L^2 norm on a specified time interval, and are used in a variety of analytical and numerical applications.

In this talk, we present an extension of an important aspect of one-dimensional analysis to spatio-spectral limiting operators in arbitrary dimensions, where one of the domains – either spatial or spectral – is a hypercube, and the other domain satisfies a symmetry condition. We estimate the distribution of eigenvalues for such operators. This is a joint work with Arie Israel.

Mishko Mitkovski, Clemson University

Title: Operator Laplacian and its applications to Toeplitz Algebras

Abstract: It was observed recently by Fulsche that techniques froui Werner's "Quantum Harmonic Analysis" (QHA) from the 80s can be used to clarify several result about Toeplitz algebras on the Bargmann-Fock space. This observation has led to a reinvigorated interest in QHA. We introduce a QHA notion of operator Laplacian to further explore this connection. We use our operator Laplacian to fully describe the Gelfand theory of radial and quasi-radial Toeplitz algebras.

This is a joint work with Vishwa Dewage.

Abdon Moutinho, Georgia Tech

Title: On the collision of two kinks for the ϕ^6 model with low speed

Abstract: We present our results in the study of the collision between two solitons known as kinks having an incoming low-speed v for a non-integrable nonlinear wave equation in dimension 1+1 known as the phi6 model. More precisely, we describe globally the solution of the PDE as a two-body system and we prove for any k in $\mathbb{N}$ that if the incoming speed v of the two solitons is small enough, then, after the collision, the two kinks move away with a velocity v_f such that $|v_f - v| < v^k$ and the energy of the remainder will also be smaller than v^k.

Stephanie Mui, Georgia Tech

An overview on Minkowski problems and the L^p dual Minkowski problem for absolutely continuous data

The classical Minkowski problem asks about the existence of a convex body with prescribed surface area measure. The same question asked for other geometric measures is known as the field of Minkowski-type problems. In the case of given absolutely continuous measures, these problems are equivalent to some Monge-Ampere type PDE's. The L^p dual curvature measure was introduced by Lutwak, Yang, and Zhang in 2018. The associated Minkowski problem, known as the L^p dual Minkowski problem, asks about existence of a convex body with prescribed L^p dual curvature measure. Among its important cases are the classical Minkowski problem, the Aleksandrov problem, and the log Minkowski problem. I will discuss some recent results in this problem as well as give an overview of the progress made for other related Minkowski problems.

Eyvindur Palsson, Virginia Tech

Title: Distance problems and their many variants

Abstract: Two classic questions - the Erdos distinct distance problem, which asks about the least number of distinct distances determined by points in the plane, and its continuous analog, the Falconer distance problem - both focus on distance. Here, distance can be thought of as a simple two point configuration. Questions similar to the Erdos distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as triangles, which can be viewed as three point configurations. In this talk I will go through some of the history of such point configuration questions and end with some recent results.

Alexei Poltoratski, University of Wisconsin-Madison

Title: Pointwise convergence of the scattering data

Abstract: It is well known that the scattering transform for second order differential equations can be viewed as a non-linear version of the classical Fourier transform. This connection brings up several natural questions on the non-linear analogs of the classical theorems of Fourier analysis. In this talk we will discuss an analog of Carleson's theorem on pointwise convergence of the Fourier transform in non-linear settings.

Eli Putterman, Tel Aviv University

Title: Small-ball probabilities for mean widths of random polytopes

The classical theory of random polytopes addresses questions such as computing the expectation or variance of geometric parameters associated to a random polytope (e.g., volume, number of facets, or mean width); more recent theory also aims to obtain concentration of measure for such quantities. The new theory of higher-order projection bodies naturally leads to a question in random polytopes which current theory, surprisingly, does not address: bounding a high negative moment of the mean width of a certain random polytope, which requires bounding the probability that this mean width is a small fraction of its expectation ("small-ball estimates"). These small-ball estimates use different tools from those commonly employed in the field of random polytopes, and it turns out that the behavior of the negative moment demonstrates a phase transition. We will conclude by mentioning some related open problems.

Kevin Ren, Princeton University

Title: Sharp Furstenberg Sets Estimate in the Plane

Abstract: Fix a real number $0 < s <= 1$. A set E in the plane is a s-Furstenberg set if there exists a line in every direction that intersects E in a set with Hausdorff dimension s. For example, a planar Kakeya set is a special case of a 1-Furstenberg set, and indeed we know that 1-Furstenberg sets have Hausdorff dimension 2. However, obtaining a sharp lower bound for the Hausdorff dimension of s-Furstenberg sets for any $0 < s < 1$ has been a challenging open problem for half a century. In this talk, I will explain the recent resolution of the Furstenberg set conjecture using tools from Fourier analysis and additive combinatorics. Joint works with Yuqiu Fu and Hong Wang.

Sasha Reznikov, Florida State University

Title: Optimization problems and Geometric Measure Theory

Abstract: We will talk about the optimal covering and packing problems on sets that fall in the framework of the Geometric Measure Theory. Examples of such sets are rectifiable (but not smooth) sets on one end of the spectrum, and purely unrectifiable sets (e.g., fractals) on the other side of the spectrum. Although the methods for treating these examples are significantly different, some assumptions surprisingly appear in both cases. We will discuss the best known results as well as some conjectures for the future research.

Paul Simanjuntak, Texas A&M

Title: An unified probabilistic approach to Lp affine isoperimetric inequalities

Abstract: In the class of convex sets, the isoperimetric inequality admits several stronger affine formulations. These involve fundamental geometric constructions of Busemann and Petty, which were subsequently placed in a functional analytic framework by Lutwak and Zhang. Specifically, they introduced Lp centroid bodies, $p \geq 1$, and established a family of affine isoperimetric inequalities that includes the Blaschke-Santalo inequality. Along this line of work, Lutwak raised the question of

encompassing the non-convex (star-shaped) range, i.e. when p¡1, which naturally includes the Busemann intersection inequality. I will discuss an unified probabilistic approach that uses a new representation of star-shaped sets, expressing them as a special average of convex sets. Based on works with R. Adamczak, G. Paouris, and P. Pivovarov.

Mariana Smit Vega Garcia, Western Washington University

Title: Almost minimizers of a lower-dimensional free boundary problem.

Abstract: In this talk, we will consider almost minimizers of a two-phase free boundary problem given by an energy functional connected to the fractional Laplacian. We prove regularity of almost minimizers, and show that the two free boundaries cannot touch. Finally, we will discuss the regularity of the free boundary. This is joint work with Mark Allen.

Brandon Sweeting, University of Alabama

Title: Multiplier Weak-Type Inequalities for Maximal Operators and Singular Integrals

Abstract: We discuss a kind of weak type inequality for the Hardy-Littlewood maximal operator and Calderón-Zygmund singular integral operators that was first studied by Muckenhoupt and Wheeden and later by Sawyer. This formulation treats the weight for the image space as a multiplier, rather than a measure, leading to fundamentally different behavior; in particular, as shown by Muckenhoupt and Wheeden, the class of weights characterizing such inequalities is strictly larger than A_p. In this talk, I will discuss quantitative estimates obtained for A_p weights, $p > 1$, that generalize those results obtained by Cruz-Uribe, Isralowitz, Moen, Pott and Rivera-Ríos for $p = 1$, both in the scalar and matrix weighted setting. I will also discuss an endpoint result for the Riesz potentials as well as recent work on the characterization of such weights.

Krystal Taylor, the Ohio State University

Title: Prescribed Projections and Kakeya sets in a nonlinear setting

Abstract: Davies' efficient covering theorem states that an arbitrary measurable set in the plane can be covered by full lines so that the measure of the union of the lines has the same measure as the set. This result has an interesting dual formulation in the form of a prescribed projection theorem. In this talk, we formulate each of these results in a nonlinear setting. As an application, we consider how duality between curves and points can be used to construct nonlinear Kakeya sets.

Konstantin Tikhomirov, Carnegie Melon

Title: On pseudospectrum of inhomogeneous non-Hermitian Gaussian matrices

Abstract: We consider the problem of estimating the smallest singular value of random matrices G-zI, where G is an inhomogeneous Gaussian matrix. Our method is based on a recursive estimate in terms of the smallest singular values of submatrices and does not rely on covering arguments.

Manasa Vempati, University of Louisiana

Title: Weighted estimates of Singular Integral operators.

Abstract: We will characterize weights, so two weight estimates for singular Integral operators hold in the generality of spaces of homogeneous type with some applications.

Sasha Volberg, Michigan State University

Title: Learning big matrices via harmonic analysis on Boolean cube and on cyclic groups.

Abstract: The interaction between learning theory and harmonic analysis was emphasized by mathematics of quantum computing. One of the outstanding open problem in this area concerns the sharp estimates in Bohnenblust–Hille inequality that generalizes a celebrated Littlewood's 4/3 lemma.

How to learn (with small error and with large probability) a complicated function or a very large matrix in a relatively small number of random (quantum) queries? Of course there should be some Fourier type restrictions on a function (a matrix) in question.

We show a dimension free Remez inequality and its application to learning big matrices that have low degree in Heisenberg-Weyl basis.

Brett Wick, Washington University in St. Louis

Title: Wavelet Representation of Singular Integral Operators

Abstract: In this talk, we'll discuss a novel approach to the representation of singular integral operators of Calderón-Zygmund type in terms of continuous model operators. The representation is realized as a finite sum of averages of wavelet projections of either cancellative or noncancellative type, which are themselves Calderón-Zygmund operators. Both properties are out of reach for the established dyadic-probabilistic technique. Unlike their dyadic counterparts, our representation reflects the additional kernel smoothness of the operator being analyzed. Our representation formulas lead naturally to a new family of T1 theorems on weighted Sobolev spaces whose smoothness index is naturally related to kernel smoothness. In the one parameter case, we obtain the Sobolev space analogue of the A_2 theorem; that is, sharp dependence of the Sobolev norm of T on the weight characteristic is obtained in the full range of exponents. As an additional application, it is possible to provide a proof of the commutator theorems of Calderón-Zygmund operators with BMO functions.

Ashley Zhang, Vanderbilt

Title: Complex analytic approach to spectral problems for differential operators

Abstract: This talk will be about applications of complex function theory to spectral problems for canonical systems, which constitute a broad class of second order

differential equations. I will start with the basics of Krein-de Branges theory. Then I will present an explicit algorithm for inverse spectral problems developed by Makarov and Poltoratski for locally-finite periodic spectral measures, as well as an extension of their work to certain classes of non-periodic spectral measures. Finally, I will talk about some recent developments on direct spectral problems. This is partially based on joint work with Alexei Poltoratski.

Artem Zvavitch, Kent State University

Title: Weighted Brunn-Minkowski Theory.

Abstract: The Brunn-Minkowski Theory concerns the behavior of convex bodies in $\mathbb{R}^n$ (compact, convex sets with non-empty interior), by studying their properties e.g. volume, surface area, projections, and Minkowski sum. We shall discuss a generalization of this theory to the measure theoretic setting (replacing volume with some Borel measure with density). In particular, we defined the mixed measures of three convex bodies, and present an integral formula for such mixed measures. We will show inequalities for this quantity, such as Minkowski's First and Second inequality and as well as Fenchel's inequality. As applications, we study log-submodularity and supermodularity of the measure of Minkowski sums of symmetric convex bodies, inspired by recent investigations of these properties for the volume. In particular, we will verify that a Radon measure with the supermodularity property must be the Lebesgue measure.

This is a part of a joint project(s) with Matthieu Fradelizi, Dylan Langharst and Mokshay Madiman.