1. Prove that the zeros of the polynomial \( P(z) = z^7 + 7z^4 + 4z + 1 \) lie inside the disk of radius 2 centered at the origin.

2. Let \( A \) and \( B \) be \( 2 \times 2 \) matrices with real entries satisfying \((AB - BA)^n = I_2\) for some positive integer \( n \). Prove that \( n \) is even and \((AB - BA)^4 = I_2\).

3. There are given \( 2n + 1 \) real numbers, \( n \geq 1 \), with the property that whenever one of them is removed, the remaining \( 2n \) can be split into two sets of \( n \) elements that have the same sum of elements. Prove that all the numbers are equal.

4. Let \( A, B \) be \( 2 \times 2 \) matrices with integer entries, such that \( AB = BA \) and \( \det B = 1 \). Prove that if \( \det (A^3 + B^3) = 1 \), then \( A^2 = 0 \).

5. Let \( p \) be a prime integer. Prove that the determinant of the matrix
\[
\begin{pmatrix}
x & y & z \\
x^p & y^p & z^p \\
x^{p^2} & y^{p^2} & z^{p^2}
\end{pmatrix}
\]
is congruent modulo \( p \) to a product of polynomials of the form \( ax + by + cz \), where \( a, b, c \) are integers. (We say two integer polynomials are congruent modulo \( p \) if corresponding coefficients are congruent modulo \( p \).)

6. Find a nonzero polynomial \( P(x, y) \) such that \( P([a], [2a]) = 0 \) for all real numbers \( a \). (Note: \([v] \) is the greatest integer less than or equal to \( v \).)

7. Show that the curve \( x^3 + 3xy + y^3 = 1 \) contains only one set of three distinct points, \( A, B, \) and \( C \), which are vertices of an equilateral triangle, and find its area.