1. Given a probability space \((\Omega, \mathcal{F}, P)\), and a pair of Borel measurable functions \(f, g : \Omega \to \mathbb{R}\), such that \(f = g\) almost everywhere with respect to \(P\), show that \(\int f(\omega) \, dP(\omega) = \int g(\omega) \, dP(\omega)\). (please follow all the necessary steps from the definition of integral)

2. Let \(X\) be a Gaussian random variable. Please estimate from above and below \(P(X \leq 10)\).

3. Given a probability space \((\Omega, \mathcal{F}, P)\), and a random variable \(X\) with bounded fifth moment, prove that for \(t > 0\)
   \[
P(|X| < t) \geq 1 - \frac{\mathbb{E}|X|^5}{t^5}.
\]

4. Let \(X\) be a random variable taking values on the interval \([1, 2]\). Find sharp lower and upper estimates on the quantity \(\mathbb{E}X \cdot \mathbb{E} \frac{1}{X}\). Provide an example of a random variable for which the lower estimate is attained. Provide an example of a random variable for which the upper estimate is attained.

**Hint.** For the lower bound, justify and use the inequality
   \[
a b \leq \frac{1}{2} \left( \frac{a}{2} + b \right)^2.
\]