TEST 2, PROBABILITY I

Select 2 problems out of 3.

1. Show that if $X_1, ..., X_n, ...$ are independent and have characteristic function $e^{-|t|^\alpha}$, then $\frac{X_1 + ... + X_n}{n^{\frac{1}{\alpha}}}$ has the same distribution as $X_1$.

2. Show that if $X$ and $Y$ are independent and $X + Y$ and $X$ have the same distribution, then $Y = 0$ a.s.
   Hint: use characteristic functions.

3. Suppose $X_i$ are i.i.d. with $X_i \geq 0$, $\mathbb{E}X_i = 1$ and $\sigma^2 = 1$. Show that $2(\sqrt{X_1 + ... + X_n} - \sqrt{n})$ converges weakly to the standard normal distribution.
   Hint: Use the CLT and the definition of weak convergence (and don’t forget that $F_X(x) = P(X \leq x)$).