This exam contains 12 pages (including this cover page) and 13 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

Instructions:

- You are required to show your work and justify your answers for all questions except where explicitly stated.
- Organize your work, in a reasonably neat and coherent way.
- Calculators, cell phones, books and notes are not allowed.

Academic integrity is expected of all Georgia Institute of Technology students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination. Please sign below to indicate that you have read and agree to these instructions.

______________________________
Signature of student

For official use only:

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1. (4 points) Line \( L \) passes through point \( P(0, 0, 1) \), and is perpendicular to the plane \( x + y + z = 4 \). Does \( L \) pass through the plane \( x + 2y + z = 4 \)? If so, find the point of intersection. If not, explain why.

2. (4 points) Evaluate the limit or show that it does not exist.

\[
\lim_{{(x,y)\to(0,-2)}} \frac{xy^5}{y + 2}
\]
3. (4 points) An object is moving along the curve \( \vec{r} = 6 \sin(2t)\hat{i} + 6 \cos(2t)\hat{j} + 5t\hat{k} \), \( t \in [0, \pi] \). Find the length of the curve.

4. (4 points) An object is moving along a path in the \( yz \)-plane. At point \( P \), the tangent vector to the path is \( \vec{T} = \hat{k} \). What could the normal vector be equal to at \( P \)?
5. (8 points) Consider the set of all points that are equidistant from the line \( \langle t, 0, 1 \rangle \) and the plane \( z = 2 \).

a) Construct an equation that represents the surface.

b) Briefly describe the surface in words.

c) Give a rough sketch of the surface. Label the coordinate axes.
6. Let $g(x, y) = e^{-x} \sqrt{y + 2}$.

(a) (2 points) State the domain of $g(x, y)$, and state whether the domain is bounded or unbounded.

(b) (5 points) Sketch the level curves $C = g(x, y)$ for $C = -1$, $C = 0$, and $C = 1$, if possible. If any of these values of $C$ are not possible, state why.
7. (6 points) Let \( f(x, y) = x^3 - 9x + y^2 - 4y \). Find and classify the critical points of \( f(x, y) \).
8. (8 points) Use Lagrange Multipliers to find all points on the surface $xyz = 1$ that are closest to the origin.
9. (6 points) $D$ is the region in the $xy$-plane bounded by the curves $x + 2y = 0$ and $x + (y - 1)^2 = 1$.

(a) Construct a double integral that represents the area of $D$. Use the order of integration $dx \, dy$.

(b) Change the order of integration to $dy \, dx$. 
10. (6 points) A thin plate, whose density is proportional to the distance to the $x$-axis, lies outside the polar curve $r = 2 \sin \theta$ and inside the polar curve $r = 2 + 2 \sin \theta$.

(a) Construct an integral that represents the mass of the plate.

(b) Construct the integrals you need to find $\bar{y}$, which is the $y$-coordinate of the center of mass.
11. (7 points) Consider the triple integral
\[ \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{4-y^2}}^{1} \int_{0}^{2-x} f(x, y, z) \, dz \, dx \, dy \].

(a) Change the order of integration to \( dz \, dy \, dx \).

(b) Convert the integral to cylindrical coordinates.
12. (8 points) A solid whose density is proportional to the distance to the origin, is bounded below by the surface $x^2 + y^2 + (z - 2)^2 = 4$ and bounded above by the surface $z = \sqrt{x^2 + y^2}$.

(a) Convert $x^2 + y^2 + (z - 2)^2 = 4$ to spherical coordinates.

(b) Using spherical coordinates, construct a triple integral that represents the mass of the solid.
13. (8 points) Construct the integrals needed to compute the average value of 
\( f(x, y) = xy \) over the region in the first quadrant bounded by the curves \( y = 2x, \) 
\( y = 3x, \) \( xy = 1, \) and \( xy = 3. \) When constructing your integrals, use an appropriate 
transformation of the form \( u = f(x, y), \) and \( v = g(x, y). \)