Math 2550 Learning Objectives, Summer 2016

Learning objectives are concise statements that articulate what students are expected to be able to do upon completion of a course, or upon the completion of a section within a course. In this course there are section-level objectives that pertain to a specific section of our textbook, and course-level objectives that pertain to the entire course. This document also lists some of the pre-requisite concepts that you will need throughout this course.

Course-Level Objectives
Throughout this course, it is expected that students will do the following.

• **Construct** mathematical expressions involving vector and multivariable functions, use them to **compute** and **interpret** mathematical quantities.
• **Write** logical progressions of precise statements to justify and communicate mathematical reasoning.
• **Apply** multivariable calculus concepts to solve real-world problems.

Section-Level Objectives
By the conclusion of this course, it is expected that students will be able to do the following.

Chapter 12

1. Apply dot and cross products to interpret geometric relationships between vectors and to calculate areas and volumes (12.4).
2. Construct equations for lines and planes; use such equations to compute mathematical quantities such as points of intersection, angles, and distances; and interpret your results to describe relationships between points, lines, and planes (12.5). Example:
   - Line $L$ passes through $P(0, 0, 1)$ and is perpendicular to the plane $x+y+z=4$. Does $L$ pass through the plane $x+2y+z=4$? If so, find the point of intersection. If not, explain why.
3. Given an equation or condition that defines a quadratic surface in three variables, identify the corresponding surface and create a rough sketch of it (12.6). Example:
   - Consider the set of all points that are equidistant from the line $(t, 0, 1)$ and the plane $z = 2$. Construct an equation that represents the surface, name the surface, and create a rough sketch of it.

Chapter 13

1. Characterize motion of an object, in parametric form, in terms of its velocity, speed, acceleration, unit tangent vector (13.1).
2. Create a rough sketch of a parametric curve (13.1).
3. Differentiate sums, cross products, and dot products of vectors (13.1).
4. Integrate vector functions to describe the path of projectiles (13.2).
5. Determine the length of a curve and characterize the motion of an object in terms of its arc length parameter (13.3).
6. In two or three dimensions, characterize the motion of an object, or the shape of a curve, using curvature, the osculating circle, and the tangent and normal vectors (13.4).
7. Construct vector parametric representations of lines and curves in three dimensions (Chapter 13). Example:
• Construct a parametric vector equation that represents the intersection of the surfaces given by \( z = (4 - x^2 - y^2)^{1/2} \) and \( y^2 + x^2 - 2y = 0 \).

**Chapter 14**

1. Identify and sketch the domain and range of a function. Determine whether a region is open or closed (or neither), identify the boundary of a region, and whether the region is bounded or unbounded (14.1).
2. Sketch and describe the level curves/surfaces of a function (14.1).
3. Construct a function that has a particular domain, boundary, and/or set of level curves (14.1).
4. Show that limits of multivariable functions do not exist (using polar coordinates and/or different paths) (14.2).
5. Identify where multivariable functions are continuous and re-define functions so that they are continuous at a point (14.2).
6. Compute partial derivatives of multivariable functions, interpret them, and apply the Mixed Derivative Theorem (14.3).
7. Differentiate composite functions and interpret your results (14.4).
8. Compute gradients and directional derivatives, provide geometric interpretations of them, and describe their relationships to level curves and surfaces (14.5).
9. Find equations of tangent planes and normal lines, approximate functions how they change using tangent planes and differentials (14.6).
10. Locate and classify critical points of surfaces (14.7).
11. Solve constrained optimization problems, in two or three variables with at most one constraint (14.8).
12. Compute quadratic and cubic approximations to a function of two variables about the origin (14.9).

**Chapter 15**

1. Construct and evaluate double integrals that represent an area, or volume, or average value, for a region bounded by a set of curves or surfaces in Cartesian or polar coordinates (15.1 to 15.5).
2. Sketch the region of integration for a double integral given in Cartesian or polar coordinates (15.1 to 15.5).
3. Change the order of integration of a double or triple integral (15.1 to 15.5).
4. Construct and evaluate integrals that represent physical quantities, such as total mass, center of mass, and the centroid of a region (15.6).
5. Construct and evaluate triple integrals in spherical and cylindrical coordinates that represent physical quantities such as total mass, or a center of mass (15.7).
6. Convert a triple integral from one coordinate system to another (15.7).
7. Identify and apply a change of variable to evaluate a double integral (15.8).

**Select Prerequisite Concepts**

The only techniques of integration, for integrating a function of one variable that students need to be familiar with for quizzes, midterms, and the exam are trigonometric and algebraic substitutions (5.5 and 8.1), integration by parts (8.2), and trigonometric integrals (8.3). You are welcome to other methods if you find them helpful. Some of the linear algebra concepts that we will use include solving \( 2 \times 2 \) and \( 3 \times 3 \) linear systems, and the computation and interpretation of vector projections and determinants of \( 3 \times 3 \) matrices.