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MATH 2550
Summer 2016
Midterm 1

This exam contains 7 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

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1. (8 points) Surface $S$ is the set of all points that are equidistant from the plane $y = 0$ and the point $P(0,2,1)$.

(a) Construct an equation that represents $S$.

\[
|y| = \sqrt{x^2 + (y-2)^2 + (z-1)^2} \\
\Rightarrow y^2 = x^2 + (y-2)^2 + (z-1)^2 \\
\Rightarrow x^2 + y^2 - 4y + 4 + (z-1)^2 \\
\Rightarrow y = \frac{1}{4} \left( x^2 + (z-1)^2 \right) + 1
\]

(b) Name the surface.

paraboloid

(c) In one graph, sketch $S$ and point $P$. Label the coordinate axes.
2. (5 points) An object is moving in the $xy$-plane along the curve shown in the graph. The direction of motion is indicated with arrows. Identify the point where the curvature is maximum. Sketch the normal vector and the tangent vector at that point.

3. (6 points) Consider the two lines, $L$ and $M$.

Line $L$ passes through $P(1, -2, 2)$ and is parallel to vector $(1, 2, 1)$.

Line $M$ passes through $Q(12, -1, 1)$ and is parallel to vector $(3, -1, -1)$.

(a) Do the lines intersect?

Let $L = \left[ \frac{1}{2} \right] + \left[ \frac{1}{2} \right] t$, $M = \left[ \frac{12}{3} \right] + \left[ \frac{-1}{-1} \right] u$

Set $L(t) = M(u)$, solve for $t$ and $u$.

\[
\begin{align*}
1 + t &= 12 + 3u \\
-2 + 2t &= -1 - u \\
2 + t &= 1 + u
\end{align*}
\]

\[t = 2, \ u = -3\]

\[\Rightarrow \text{consistent system} \Rightarrow \text{lines intersect}\]

(b) If the lines intersect, where do they intersect? If they do not intersect, compute the distance between them.

\[\left( 3, 2, 4 \right)\]
4. (8 points) An object is moving along the helix
\[ \mathbf{r}(t) = 2\sin(2\pi t)\mathbf{i} + 2\cos(2\pi t)\mathbf{j} + 3\pi t\mathbf{k}, \quad t \geq 0 \]

Find all values of \( t \) when the distance along the curve, from the object to point \( P(0, 2, 3\pi) \), is equal to \( 5\pi \).

\[ \left| \frac{\mathbf{r}}{\sqrt{3}} \right| = \text{Speed} = \sqrt{\left( \frac{4\pi}{\sqrt{3}} \cos(2\pi t) \right)^2 + \left( \frac{4\pi}{\sqrt{3}} \sin(2\pi t) \right)^2 + (3\pi)^2} \]

\[ = \sqrt{\frac{4\pi^2}{3} + \frac{4\pi^2}{3} + 9\pi^2} \]

\[ = 5\pi \]

\[ S = \int_1^t 5\pi \, dt, \quad \text{because } t = 1 \text{ at } P \]

\[ = 5\pi (t-1) \]

we need \( |S| = 5\pi \), or \( 5\pi = 15\pi t - 5\pi \)

\[ \Rightarrow 5\pi = \pm (5\pi t - 5\pi) \]

\[ \Rightarrow t = 0, 2 \]

only \( \square \) for only finding \( t = 2 \)
5. (8 points) A golfer sends a ball from a tee towards a pin, with a speed of $6\sqrt{2}$ m/s at an angle of 45° from the ground. The ball lands somewhere on top of a small hill that is 1 m high. If the pin does not get in the way, where does the ball land in relation to the pin? You can use $g = 10$ m/s².

\[ \vec{s} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \]

\[ \vec{v}_0 = \begin{bmatrix} c_x \\ -10t + c_x \end{bmatrix} \]

\[ \vec{v}_0(0) = \begin{bmatrix} 6 \\ 6 \end{bmatrix} \Rightarrow \vec{v}(t) = \begin{bmatrix} 6 \\ 6 - 10t \end{bmatrix} \]

\[ \vec{r} = \int \vec{v} \, dt = \begin{bmatrix} 6t + c_1 \\ 6t - 5t^2 + c_2 \end{bmatrix} \]

Set \( \vec{r}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) \Rightarrow \( \vec{r} = \begin{bmatrix} 6t \\ 6t - 5t^2 \end{bmatrix} \)

At top of hill, \( 6t - 5t^2 = 1 \)

\[ 5t^2 - 6t + 1 = 0 \]

\[ (5t - 1)(t - 1) = 0 \]

\[ t = 1, \frac{1}{5} \]

when \( t = 1 \), horizontal range is 6 meters

\( \Rightarrow \) ball lands before pin
6. (9 points) An object is moving along the curve \( y = (t - 1)^2 \) in the direction of increasing \( t \), for \( t \geq 0 \).

(a) Compute the curvature of the curve for all \( t \geq 0 \).

\[
K = \frac{|y''|}{(1 + (y')^2)^{3/2}}
\]

\[
= \frac{2}{(1 + (2t-2)^2)^{3/2}}
\]

(b) Construct the equation of the osculating circle when \( t = 1 \).

\[
\text{radius of circle} = \frac{1}{K(1)} = \frac{1}{2(1+0)^{3/2}} = \frac{1}{2}
\]

\[
(t-1)^2 + (y - \frac{1}{2})^2 = \frac{1}{2^2}
\]

(c) Create a rough sketch of the curve and the osculating circle when \( t = 1 \).
7. (6 points) A parallelogram has vertices $A(2, -1, -1)$, $B(1, 0, -1)$, $C(1, -1, 3)$ and $D$. Vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ point in the same direction.

(a) Find the coordinates of $D$.

$$\overrightarrow{AB} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{AB}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \Rightarrow D(0, 0, 3)$$

(b) Find the area of the parallelogram.

$$\text{area} = \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|$$

$$= \left| \begin{pmatrix} i & j & k \\ -1 & 1 & 0 \\ -1 & 0 & 4 \end{pmatrix} \right|$$

$$= \left| \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix} \right| = \sqrt{33}$$

**Bonus Questions**

1. (1 mark) What is a learning objective?

   A statement that articulates what students are expected to do

2. (1 mark) An object is moving along the curve $y = y(t)$, for $t \geq 0$. Why is the curvature of the curve equal to zero at the inflection points of $y(t)$?

   Because at inflection points $y'' = 0$, and

   $$\kappa(t) = \frac{|y''|}{(1 + (y')^2)^{3/2}} = \frac{0}{(1 + (y')^2)^{3/2}} = 0$$
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MATH 2550

Summer 2016
Midterm 2

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1. (4 points) Below is a contour map for a function $f(x, y)$. At points P and Q, sketch the normalized gradient vector

$$\vec{g}(x, y) = \frac{\nabla f}{|\nabla f|}$$

2. (6 points) Is there a direction in which the rate of change of $f(x, y) = x^2y + xy^2$ at $P(1, 2)$ equals 10? If so, identify the direction. If not, explain why.

$$\nabla f = \begin{bmatrix} 2xy + y^2 \\ x^2 + 2xy \end{bmatrix}, \quad \nabla f(1, 2) = \begin{bmatrix} 8 \\ 5 \end{bmatrix}, \quad |\nabla f| = \sqrt{8^2 + 5^2}$$

$$|\nabla f| < 10$$

$\Rightarrow$ no direction in which rate of change is 10
3. (2 points) Compute the rate of change of $z = 2x^2 + 3y^2 + 5$, in the $x$-direction, at the point $P(2, 2, 25)$.

\[ \frac{\partial z}{\partial x} = 4x + 0 + 0 \]

\[ \frac{\partial z}{\partial x} \bigg|_{(2,2,25)} = 8 \]

4. (6 points) A closed rectangular box that is 2 cm long, 3 cm wide, and 5 cm high, is covered by a coat of paint $\frac{1}{2}$ cm thick. Use a differential to estimate the amount of paint on the box.

\[ V = x \cdot y \cdot z \]

\[ dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \]

\[ = yz \, dx + xz \, dy + xy \, dz \]

use $dx = dy = dz = 2 \cdot \frac{1}{2} = 1$

use $x = 2$, $y = 3$, $z = 5$

\[ dV = 3 \cdot 5 \cdot 1 + 2 \cdot 5 \cdot 1 + 2 \cdot 3 \cdot 1 \]

\[ = 3 \]

\[ = 3 \]
5. Consider the function \( f(x, y) = \ln (x^2 + y^2 - 4) \).

(a) (3 points) State the domain of \( f(x, y) \), and sketch the domain of \( f(x, y) \).
\[
\begin{align*}
  x^2 + y^2 &> 4 \\
\end{align*}
\]

(b) (1 point) State the boundary of the domain.
\[
\begin{align*}
  x^2 + y^2 &= 4 \\
\end{align*}
\]

(c) (1 point) State whether the domain is bounded or unbounded.

UNBOUNDED

(d) (1 point) State whether the domain is open and/or closed.

OPEN

(e) (1 point) State the range of \( f(x, y) \).

\( \mathbb{R} \)

(f) (3 points) Sketch the level curves \( C = f(x, y) \), for \( C = -1 \), \( C = 0 \) and \( C = 1 \), if possible. If it not possible to sketch the level curves for any of these values of \( C \), explain why.

\[
\begin{align*}
  C &= \ln(x^2 + y^2 - 4) \\
  \Rightarrow \quad x^2 + y^2 &= 4 + e^C
\end{align*}
\]
6. (6 points) Suppose the temperature, \( T \), of a moving object, is a function of time, \( t \), its \( x \)-coordinate, its \( y \)-coordinate, and \( z \)-coordinate, so that
\[
T = T(x, y, z, t)
\]

The \( x \), \( y \), and \( z \) coordinates are given by
\[
x(t) = t^2 - 2t + 1, \quad y(t) = \cos(\pi t) + e^{-t}, \quad z(t) = 2t
\]

When \( t = 1 \),
\[
\frac{\partial T}{\partial x} = 4, \quad \frac{\partial T}{\partial y} = -2, \quad \frac{\partial T}{\partial z} = 2, \quad \frac{\partial T}{\partial t} = -2.
\]

When \( t = 1 \), is the temperature of the object increasing or decreasing?

\[
\frac{dT}{dt} = \left( \frac{\partial T}{\partial x} \right) \frac{dx}{dt} + \left( \frac{\partial T}{\partial y} \right) \frac{dy}{dt} + \left( \frac{\partial T}{\partial z} \right) \frac{dz}{dt} + \left( \frac{\partial T}{\partial t} \right) \frac{dt}{dt}
\]

\[
= T_x \cdot (2 - 2) + T_y \cdot (-\sin(\pi) - e^{-1}) + T_z \cdot 2 + T_t \cdot 1
\]

\[
\frac{dT}{dt}(1) = 4(0) + (-2)(-\sin(\pi) - e^{-1}) + 2
\]

\[
= 2/e - 2 < 0
\]

\[
\Rightarrow \text{temperature decreasing}
\]
7. (8 points) Consider the surface

\[ x^2 - 4x + \frac{y^3}{3} + \frac{z^2}{2} + 4z = 0 \]

Identify all points on the surface, if any, where the tangent plane is parallel to the plane \( y = 1 \).

Let \( F(x, y, z) = x^2 - 4x + \frac{y^3}{3} + \frac{z^2}{2} + 4z = 0 \)

\[ \nabla F = \begin{bmatrix} 2x - 4 \\ y^2 \\ z + 4 \end{bmatrix} \]

Let \( G = y - 1 = 0 \) \implies \nabla G = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \)

We want points that satisfy \( \nabla F = k \nabla G \) \implies \nabla F \times \nabla G = [0]

\( \implies 2x - 4 = 0 \implies x = 2 \)

\( y^2 = k \implies y = \pm k \)

\( z + 4 = 0 \implies z = -4 \)

\( \left( 2, \pm k, -4 \right) \) \implies \( k = \pm 2 \)

\( (2, \pm 2, -4) \) \implies \( k = \pm 2 \sqrt{4}, \ (2, \pm 2 \sqrt{4}, -4) \)
8. (4 points) Evaluate the limit or show that the limit does not exist.

\[
\lim_{(x,y) \to (2,0)} \frac{y(x-2)}{(x-2)^4 + y^2}
\]

Along \( y = 0 \), \( \lim_{x \to 2} \frac{0(x-2)}{(x-2)^4 + 0} = 0 \)

Along \( y = x-2 \) \( \lim_{y \to 0} \frac{y(y)}{y^4 + 4} = \lim_{y \to 0} \frac{1}{y^2 + 4} = 1 \)

\( \Rightarrow \) limit value depends on path

\( \Rightarrow \) limit does not exist

May other ways of solving this, but student must use paths that pass through \((2,0)\)
9. (4 points) Construct an equation of the level curve for \( f(x, y) \) that passes through the point \( P(3, -1) \).

Sketch the level curve \( f(x, y) = \sqrt{x + y^2 - 3} \)

\[
f(3, -1) = \sqrt{3 + (-1)^2 - 3} = 1
\]

\[
l^2 = x + y^2 - 3
\]

\[
x = 4 - y^2
\]

Bonus Questions

1. (1 point) Give one example of a learning objective that pertains to the topics covered in this midterm.

   Many possible answers, but students must write something they are expected to describe, not a topic that is covered.

2. (1 point) If possible, give an example of a function, \( f(x, y) \), whose gradient is

\[
\nabla f = (x^2 + y) \hat{i} + (y^3 + x) \hat{j}
\]

by inspection:

\[
f = \frac{x^3}{3} + \frac{y^4}{4} + xy
\]

or: \( f = \frac{x^3}{3} + \frac{y^4}{4} + xy + C, \ C \in \mathbb{R} \)
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MATH 2550

Summer 2016

Midterm 3

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1. **(4 points)** A flat plate is in the shape of the region bounded by the lines \( x = 0, y = 2, \) and \( y = 2x \). The temperature on the plate is given by

\[
T(x, y) = 2x^2 - 4x + y^2 - 4y + 1.
\]

Identify the coldest and hottest points on the plate.

**Interior**

\[
\nabla T = \begin{bmatrix} 4x - 4 \\ 2y - 4 \end{bmatrix} = 0 \Rightarrow \begin{cases} x = 1 \\ y = 2 \end{cases}
\]

\[
D = T_{xx}T_{yy} - T_{xy}^2 = 4x^2 - 4 > 0, \quad f_{xx} > 0 \Rightarrow \min a^\top (1, 2)
\]

**Along \( x = 0 \)**

\[
T = y^2 - 4y + 1, \quad T_y = 2y - 4 = 0 \Rightarrow y = 2
\]

\[
T(0, 2) = -3
\]

**Along \( y = 2 \)**

\[
T = 2x^2 - 4x + 2^2 - 4\cdot 2 + 1 = 2x^2 - 4x - 3
\]

\[
T_x = 4x - 4 = 0 \Rightarrow x = 1
\]

\[
T(1, 2) = 2 - 4 + 4 - 4 + 1 = -1
\]

**Along \( y = 2x \)**

\[
T = 2x^2 - 4x + y^2 - 4y + 1 = 2x^2 - 4x + (2x)^2 - 4(2x) + 1
\]

\[
= 6x^2 - 12x + 1
\]

\[
T_x = 12x - 12 = 0 \Rightarrow x = 1, \quad T(1, 2) \text{ already evaluated}
\]

**Corners:**

\[
T(0, 0) = 1, \quad T(0, 2) = -3, \quad T(1, 2) = -1
\]

\[
\Rightarrow \max a^\top (0, 0), \quad \min a^\top (1, 2)
\]
2. (10 points) Use Lagrange Multipliers to calculate the minimum distance from the surface \( x^2 - y^2 - z^2 = 1 \) to the origin.

Let \( D^2 = \left( \sqrt{x^2 + y^2 + z^2} \right)^2 = x^2 + y^2 + z^2 \)

\( g = x^2 - y^2 - z^2 - 1 = 0 \)

\[ \nabla(D^2) = \lambda \nabla g \]

\[ \begin{bmatrix}
2x \\
y \\ 2z
\end{bmatrix} = \lambda \begin{bmatrix}
2x \\
-2y \\ -2z
\end{bmatrix} \]

\( \Rightarrow 2x = 2x \Rightarrow x = 0 \text{ or } \lambda = 1 \)

If \( x = 0 \), then \( 0 = x^2 - y^2 - z^2 - 1 \Rightarrow -y^2 - z^2 - 1 = 0 \) NOT POSSIBLE

If \( x \neq 0 \), then \( \lambda = 1 \). Then \( 2y = -2y \Rightarrow y = 0 \)

\( 2z = -2z \Rightarrow z = 0 \)

from constraint, \( x^2 - 0^2 - 0^2 - 1 = 0 \Rightarrow x = \pm 1 \)

\( \Rightarrow \) The points are \((\pm 1, 0, 0)\)
3. (8 points) Use Taylor’s formula for \( f(x, y) \) at the origin to construct a cubic approximation to \( f(x, y) = \sin(x^2 + y^2) \).

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<td>( f_{xyy} )</td>
<td>\text{higher order term}</td>
<td>0</td>
</tr>
<tr>
<td>( f_{yyy} )</td>
<td>\text{higher order term}</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \Rightarrow \ f \approx x^2 + y^2 \]
4. (6 points) The area of region $S$ is given by the double integral

$$
\int \int_S dA = \int_{-2}^{2} \int_{-\sqrt{4-(x-2)^2}}^{\sqrt{4-(x-2)^2}} dy \, dx
$$

(a) Create a rough sketch of region $S$.

(b) Change the order of integration to $dx \, dy$.

$$
\int_{-2}^{2} \int_{-y}^{y} dx \, dy + \int_{0}^{2} \int_{2 - \sqrt{4-y^2}}^{z} dx \, dy
$$

because $y = \sqrt{4- (x-2)^2}$

so $(x-2)^2 + y^2 = 4$

so $(x-2)^2 = -\sqrt{4-y^2}$, $x = 2 - \sqrt{4-y^2}$
5. (10 points) \( D \) is the region between the polar curves \( r_1 = -\sin \theta \) and \( r_2 = 1 - \sin \theta \).

(a) Create a rough sketch of region \( D \).

(b) Construct a double integral that represents the average value of \( f(x,y) = xy \) over \( D \).

Area bounded by \( r_1 \) is \( \pi r^2 = \pi (1)^2 = \pi \)

\[
A = \int_{\pi/2}^{2\pi} \int_{0}^{r_2} r \, dr \, d\theta - \pi
\]

Average value = \[
\frac{1}{A} \left( \int_{\pi/2}^{2\pi} \int_{0}^{r_2} r^3 \cos \theta \sin \theta \, dr \, d\theta \right)
\]
6. (6 points) Consider the region in the first octant enclosed by \( x = 2y^2 \) and \( z = 2 - x \).

(a) Construct a double integral that represents the volume of the region. Use the integration order \( dx \, dy \).

\[
\text{Volume} = \int_0^1 \int_{2y^2}^{2-x} (2-x) \, dx \, dy
\]

(b) Change the order of integration to \( dy \, dx \).

\[
\text{Volume} = \int_0^2 \int_0^{\sqrt{\frac{x}{2}}} (2-x) \, dy \, dx
\]

2) For what value of \( k \) is there a saddle at the origin? \( f = x^2 + kxy + 4y^2 \)

\[
f_{xx}f_{yy} - f_{xy}^2 = 2 - k^2 \Rightarrow k > 16 \text{ for saddle (or } |k| > 4)\]