Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 | 
2 | 
3 | 
4 | 

TOTAL 

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1 (10 points) Consider, **but do not graph**, the polar relationship

\[
\sqrt{3 + \cos^2(\theta)} - \cos(3\theta)
\]

Check the symmetries and briefly explain:

The **only symmetry is that the graph is symmetric** (even) **about the horizontal axis**. **This is because the function is unchanged when we change** \( \theta \) **to** \(-\theta\). **It is NOT unchanged when we change** \( \theta \) **to** \( \theta + \pi \) **or to** \( \pi - \theta \).

2 (10 points)

Let \( z = -8 + 8\sqrt{3}i \).

a) Find the polar coordinates of \( z \): \( r = \_16 \_ \theta = \_2\pi/3 \_ \)

b) How many numbers \( w \) are there satisfying \( w^4 = z \)?

**ANSWER:** There are **4** such numbers.

Evaluate (in rectangular coordinates \( x = x + yi \)):

c) The solutions \( w \) of \( w^4 = z \) are \( w = 2 \exp(i\pi/6 + n\pi/2) = \sqrt{3} + i, -1 + \sqrt{3}i, -\sqrt{3} - i, \) **and** \( 1 - \sqrt{3}i \).
3 (10 points) Evaluate the following, if it exists. If it is divergent, state clearly why.

a) \( \lim_{x \to 1} \frac{1 - x^2}{1 - x^3} = \) ________________________

Say....wasn't that problem 23 on p. 617? **Sure was! It is a good idea to do the homework problems. The easiest way to do this problem is to look at**

\( \lim_{x \to 1} \frac{1 - x^2}{1 - x^3} \)

and then take the square root of the answer, since the square root is a continuous function. **For this easier problem, L'Hôpital's rule gives the limit 2/3. Hence the answer is \( \sqrt[3]{2} \).**

For part b), express the decimal as an infinite sum, and use the result to write it as a quotient of whole numbers:

b) 4.234234234234... = 4.32 \times \sum_{k=0}^{\infty} \frac{10^{-3}}{1 - .001} = \frac{4230}{999} = \frac{470}{111}

Are the following convergent or divergent?

c) \( \sum_{k=0}^{\infty} (-1)^k \frac{k^{2k}}{k!2^k} \)

is divergent, because of the ratio test; \( |a_{n+1}/a_n| > 1 \) for all large \( k \).

d) \( \sum_{k=0}^{\infty} \frac{k!}{(k + 2)! + \sqrt{k}} \)

is convergent because \( a_k < 1/k^2 \), so the series can be compared from above with the convergent p-series with \( p=2 \).

4 (10 points)

a) (No credit for calculator answers in this problem, only for approximations.) Answer the gunslinger, using Taylor's polynomial of degree 2 and the knowledge that \( 72^2 = 5184 \), to estimate the square root of 5248. Also estimate the error.

\[
72 + \frac{64}{2 \cdot 72} - \frac{64^2}{8 \cdot 72^3} = 72 \frac{323}{729} \leq \sqrt{5248} \leq 72 \frac{52327}{118098} = 72 + \frac{64}{2 \cdot 72} - \frac{64^2}{8 \cdot 72^3} + \frac{64^3}{16 \cdot 72^5}
\]

Numerically, this would be \( 72.44307270 \leq \sqrt{5248} \leq 72.44308117 \). Notice that the Lagrange formula shows that the error is positive, so the left side is exactly the Taylor polynomial.
b) Write the Taylor series for \( x \) near \( a=0 \) for

\[ x \exp(-2x^3) = \sum_{k=0}^{\infty} (-2)^k \frac{x^{3k+1}}{k!} \]

c) Write the Taylor series for \( x \) near \( a=1 \) for \( x^5 \).

\[
\texttt{taylor(x^5,x=1);} \\
1 + 5 \, (x - 1) + 10 \, (x - 1)^2 + 10 \, (x - 1)^3 + 5 \, (x - 1)^4 + (x - 1)^5
\]

Yes, there are only a finite number of terms. All further coefficients are 0.