Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible.

**Important. Think about what kind of a mathematical object a question is asking for. You will receive 0 points if your answer is not the "right kind of animal."**

**SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:**

```
1 |
2 |
3 |
4 |
```

TOTAL
1 (10 points). Let
\[
\mathbf{v} = 3 \mathbf{i} - 3 \mathbf{j} + 3 \mathbf{k}, \quad \mathbf{w} = -\mathbf{i} - 2 \mathbf{j} + 3 \mathbf{k}
\]
\[
\mathbf{V} = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 0 & 1 \\ -7 & 0 & 7 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}
\]
Calculate the following, or briefly state why they are undefined:

a) The cosine of the angle between \( \mathbf{v} \) and \( \mathbf{w} \) is:
_________________________

b) \( 2\mathbf{W} - \mathbf{v} = \)
_________________________

c) \( \mathbf{W}^t \mathbf{M} \mathbf{N} = \)
_________________________

d) \( \mathbf{N} \mathbf{W} = \)
_________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________
_________________________________________________

2 (10 points). Consider the following points in space:
\[
\mathbf{O} = (0,0,0), \quad \mathbf{P} = (-1,-2,5), \quad \mathbf{Q} = (-1,0,1), \quad \mathbf{R} = (2,1,2)
\]

a) Find the equation for the plane passing through \( \mathbf{P}, \mathbf{Q}, \) and \( \mathbf{R}:
\[
\boxed{__________________________} = \boxed{__________________________}
\]

b) Find the area of the triangle \( \mathbf{PQR} \)
\[
\text{AREA} = \boxed{__________________________}
\]

KEY FORMULA OR METHOD (optional for partial credit)____________________
_________________________________________________
3 (10 points). A company called the Henergy Coop produces two grades of environmentally friendly fuel, Cheep, which is 75% gasoline and 25% poultry by-products; and Souper, which is 80% gasoline and 20% poultry by-products. Let g and b respectively denote the amounts (in kg.) of gasoline and by-products used in producing c kg. of Cheep and s kg. of Souper.

a) Find the production matrix $P$ such that

$$
\begin{bmatrix}
 b \\
 g
\end{bmatrix} = P \begin{bmatrix}
 S \\
 C
\end{bmatrix} = \begin{bmatrix}
 & & \\
 & & \\
 & & \\
\end{bmatrix} \begin{bmatrix}
 S \\
 C
\end{bmatrix}
$$

(fill in blanks)

b) Find

$$P^{-1} = \begin{bmatrix}
 & & \\
 & & \\
 & & \\
\end{bmatrix}
$$

c) If the factory has consumed 10 (metric) tons of poultry by-products and 36 tons of gasoline, how much of each fuel grade has been produced?

$$S = \__________________________$$

$$C = \__________________________$$

KEY FORMULA OR METHOD (optional for partial credit)_____________________
4 (10 points). Find all solutions of the following. Show your work.

a) \[
\begin{bmatrix}
5 & -1 & 2 \\
1 & 1 & 1 \\
7 & 1 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
6 \\
3 \\
0
\end{bmatrix}
\]

b) \[
\begin{align*}
2x + y &= 4 \\
x - y + z &= 1 \\
2x + z &= 4
\end{align*}
\]

\[
\begin{align*}
x &= \\
y &= \\
z &= 
\]

KEY FORMULA OR METHOD (optional for partial credit) __________________________