INSTRUCTIONS: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Some partial credit may be given if you have clearly indicated the method. Partial credit will not, however, be given for mathematical errors. Graphs should be clearly labeled. **Calculators are not permitted if they can display graphs**, store formulae, or do symbolic mathematics (algebra & calculus).

SCORING: Best 10 of 12 problems. You may choose to do only ten, or else to do more problems for possible extra credit to offset errors in the other ten.
PART I. Integrals and derivatives:

1. \[ \int \sin(2x) \, dx = \] ____________________________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

   \[ \int \left( 2x^3 - \frac{3}{x} \right) \, dx = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

   \[ \int \left( \frac{1}{1+x^2} \right) \, dx = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

   \[ \int_0^1 2 \sin^2(\xi) \cos(\xi) \, d\xi = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

2. \[ \int \cos^4(2x) \, dx = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

   \[ \int_0^{2} \frac{dx}{x \ln x} = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________

   \[ \int_0^{\pi} x^3 \cos(\xi) \, d\xi = \] ______________________________

   KEY FORMULA OR METHOD (optional for partial credit)______________________
3. \( \frac{d}{dx} x^5 = \) _______________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________
_________________________________________________

\( \int \frac{dx}{x - 2x^2} = \) _______________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________
_________________________________________________

\( \int \frac{x^2}{\sqrt{1-x^2}} \, dx = \) _______________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________
_________________________________________________
4. Suppose that the number of people attending the St. Patrick’s Day parade in Savannah from 1:00 to 3:00 is $10000 + 2000 \sin(\pi(t-1)/2)$, where $t$ is measured in hours and $t=0$ corresponds to 12:00. What is the average number in attendance?

THE FORMULA FOR THE AVERAGE IS

THE AVERAGE NUMBER IN ATTENDANCE IS

KEY FORMULA OR METHOD (optional for partial credit)

5.

a) Evaluate the integral $\int_{-2}^{-1} \frac{dx}{x}$. 

THE GENERAL FORMULA IS: 

THE NUMERICAL VALUE IS: 

KEY FORMULA OR METHOD (optional for partial credit)

b) Use the trapezoid rule and $\Delta x = .2$ to evaluate this integral. (Zero points for the calculator value of the function you may have discovered in part a!).

THE GENERAL FORMULA FOR THE TRAPEZOID RULE IS: 

THE NUMERICAL VALUE OF THE INTEGRAL IS: 

KEY FORMULA OR METHOD (optional for partial credit)
PART II. Differential equations.

6. The point of this problem is to model a process with a differential equation, and to solve the differential equation.

A snowball melts because of a combination of “surface effects” and “volume effects.” The surface area is important because heat and water must pass through the surface of the snowball when it melts. The volume is less important but still matters, because of such things as transport of heat within the snowball and penetration of solar radiation into the snowball.

For simplicity we suppose that the snowball is always spherical with radius $r(t)$, with $r$ measured in centimeters and $t$ in hours. The ambient temperature and the effects of light are independent of time.

Let us suppose that the volume decreases owing to surface effects at a rate equal to $\frac{1}{3}$ of the surface area. Also suppose that volume effects retard the change in volume at a rate equal to $\frac{1}{10}$ of the volume at any time. (So we assume that the volume effects act against the surface effects.)

a) Write a differential equation for the rate of change of the radius at any time $t$. The equation should involve $r$, but should not contain symbols for the volume or area.

$$ r'(t) = \frac{d}{dt} \left( \frac{1}{3} \cdot \frac{4}{3} \pi r^3 \right) $$

b) At time 0 the radius is 4 cm. Solve your differential equation:

$$ r(t) = \frac{1}{2} t^2 + C $$

c) How long does it take for the snowball to melt completely?

It takes ________ hours.

KEY FORMULA OR METHOD (optional for partial credit)______________

7. Find the general solution of

$$ y'' + y' + 2y = 0 $$

Find the particular solution of

$$ y'' + y' + 2y = 3 \exp(2t) $$ (exp(2t) is the same as $e^{2t}$)

Solve the problem

$$ y'' + y' + 2y = 3 \exp(2t), \ y(0) = 0, \ y'(0) = 0. $$

KEY FORMULA OR METHOD (optional for partial credit)______________
8. Do not attempt to find a formula for the solution of the differential equation in this problem! Suppose that
\[ y'(t) = \frac{e^2}{4} + \frac{1}{4e^2} - \frac{1}{2} \sinh^2(t) \]
(The ugly-looking constant is supposed to make things work out nicely. If you do not see why, and you write the right side as \( C - \sinh^2(t) \), that will not cost you more than a point. The numerical value of \( C \) is roughly 1.38109784554182.)

a) What are the possible values of \( \lim_{t \to \infty} y(t) \)?
(Use a graphical technique.)

b) For what initial values of \( y(0) \) does \( y(t) \) move towards each of the limits in part a)?

The initial values _________________ tend to the limit ____________.

The initial values _________________ tend to the limit ____________.

The initial values _________________ tend to the limit ____________.

The initial values _________________ tend to the limit ____________.

KEY FORMULA OR METHOD (optional for partial credit)______________

_________________________________________________

PART III. Integrals and geometry.

9. Find the length of the curve \( y = x^{3/3} + 1/4x \) from \( x=1 \) to \( x=3 \).

The integral expression for the length is ______________________________

The length of the curve is ______________________________

10. Find the centroid \((\bar{x}, \bar{y})\) of the quarter circle bounded by

\[ y = \sqrt{4 - x^2}, \ y=0, \text{ and } x=0. \]
11. This problem is to find the volume and surface area of a figure obtained by rotating the region bounded by

the x-axis
\[ y = \sqrt{9-x^2} \]
x=0
x=3

about the x-axis.

a) Suppose the figure is rotated about the x-axis. Which way do you slice the figure to calculate the volume (check one)?:
   - washers with thickness dx_____
   - washers with thickness dy_____
   - cylindrical shells with thickness dx____
   - cylindrical shells with thickness dy_____

b) The integral for this volume is:______________________________

c) The value of the volume is:______________________________

d) The integral for this surface area is:______________________________

e) The value of the surface area is:______________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________

12. Sketch the polar curve \( r = 4 + \cos(\theta) \) and find the area enclosed.

The integral expression for the area is ____________________________

The area of the curve is ____________________________

\[ In:= \text{PolarPlot}[4 + \cos[\text{Theta}],\{\text{Theta},0,2\ \Pi\}] \]

\[ Out= \]