

MATH 2401 - Harrell

Visualization in 3D

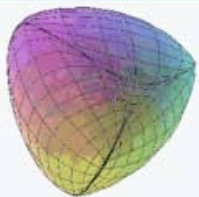
What about this class?

Math 2401 - Calculus III

http://www.math.gatech.edu/~harrell/2401/

Google

Calendar Gmail Financial Lookup Commercial News Travel Scientific Georgia Tech BugMeNot People Google Maps YouTube Wikipedia



Class page

T-Square
Assignments

Syllabus

Resources

E-mail

The Prof
TA for T1
TA for T2
TA for T3
The Archive

Prof. Harrell's Math 2401 Georgia Tech, Fall, 2008

[School of Mathematics](#)
[Skiles Building, Office 218D](#)
[Georgia Institute of Technology](#)
[Atlanta GA 30332-0160](#)

email: harrell@math.gatech.edu
Phone: (404) 894 3300

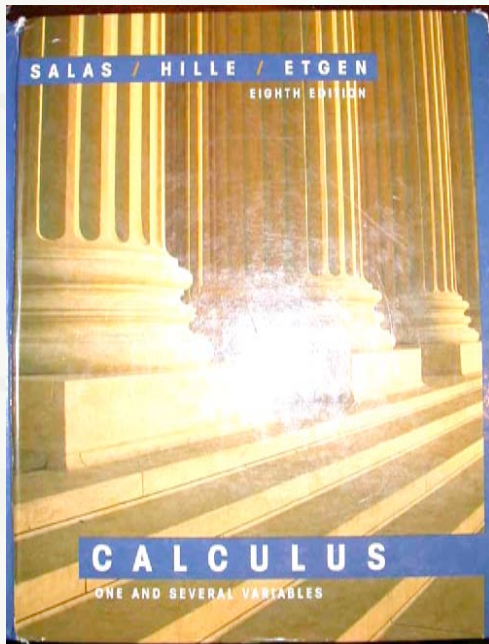
Current items of interest

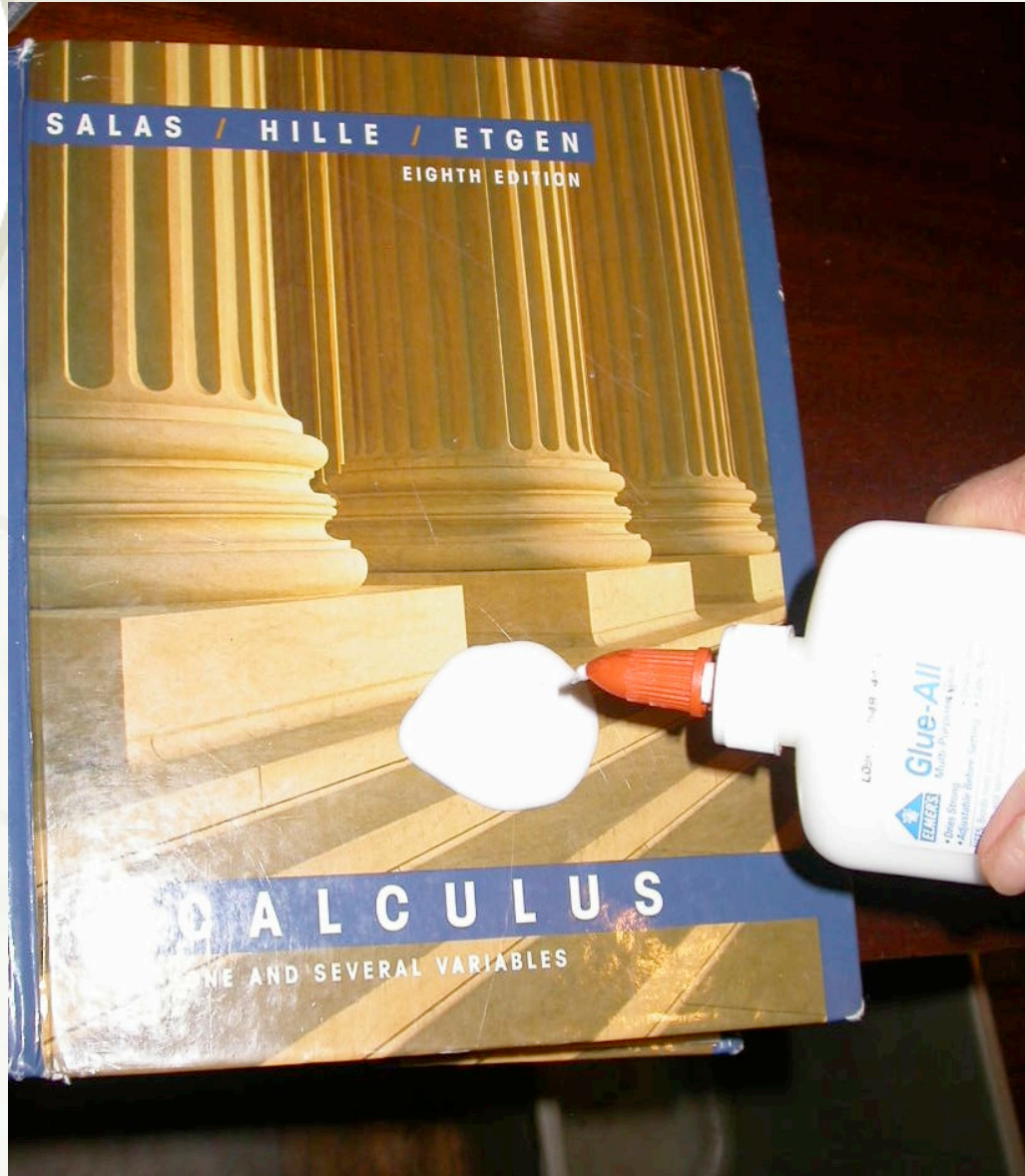
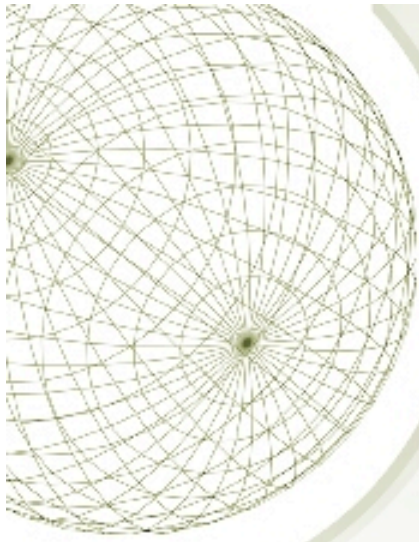
- I am working on materials for the class and looking forward to meeting my students on Monday afternoon. Meanwhile, if you want to [introduce yourself](#), please do!
- The [Syllabus](#) is ready.

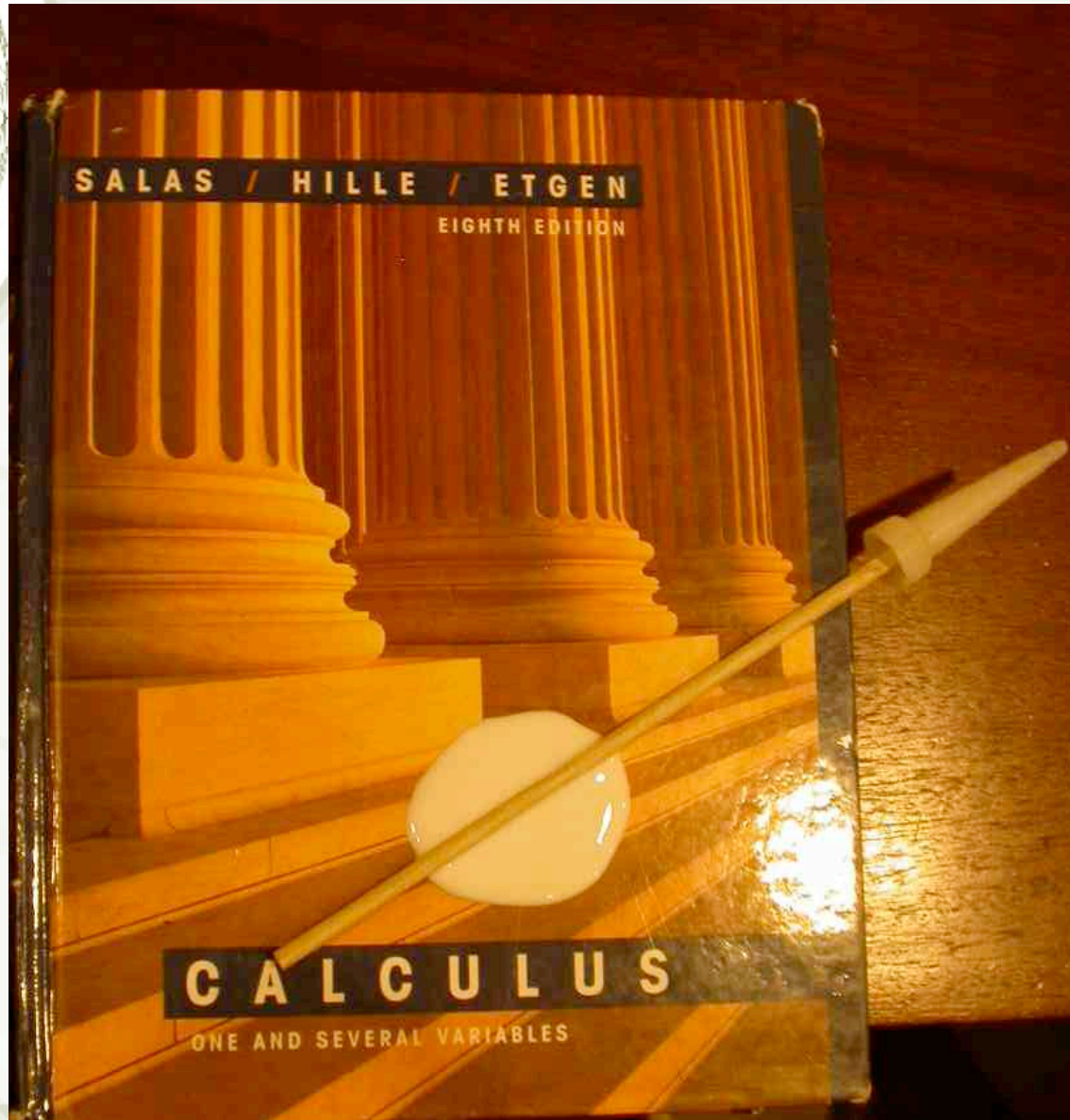
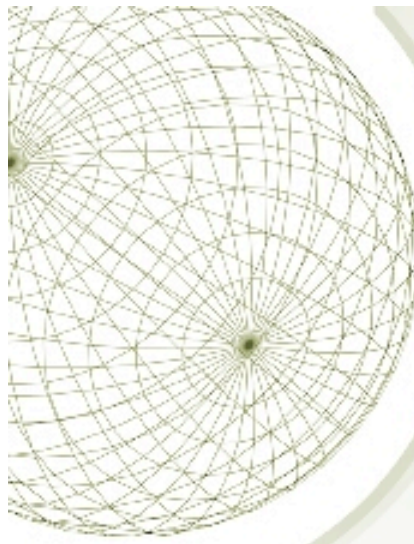
Useful resources

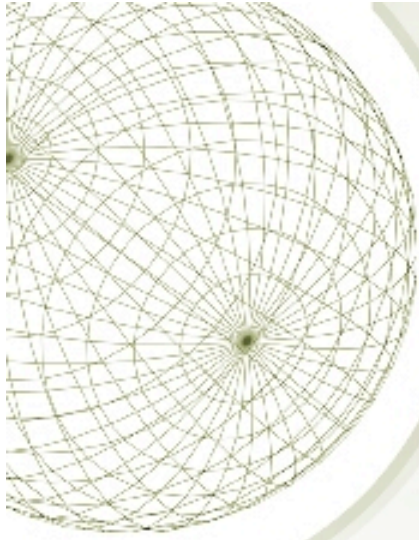
- The class [syllabus](#)
- [The School of Mathematics on-line resources](#) for the first two years of calculus. This contains links to Maple worksheets, Java applets, and other materials used in Math 2401.
- The [vector calculus page](#) at mathphysics.com
- The [generic syllabus for Calculus III](#)

*Just to see what happens,
let's glue together two ideas.*









Vectors & Functions

Vectors & Derivatives

But first - Vector Boot Camp!



Copyright 2008 by Evan L. Harrell II.

A wireframe sphere is positioned in the top-left corner of the slide. It is composed of a grid of lines forming a spherical shape, with a central point from which the lines radiate outwards.

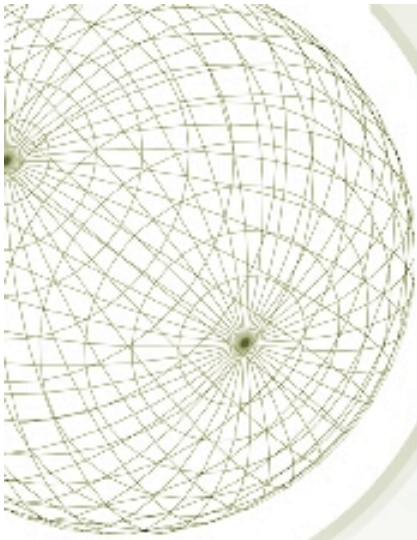
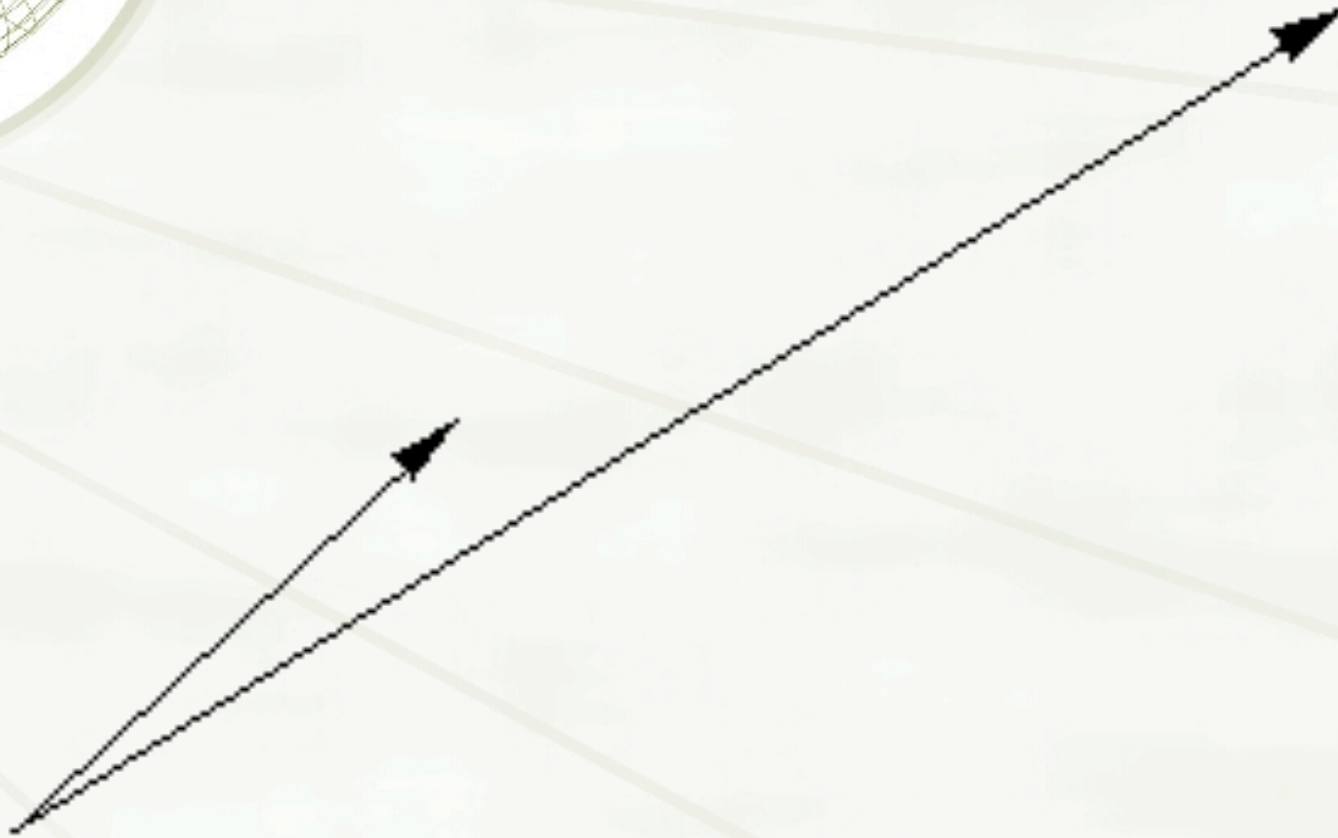
Two ways to analyze vectors

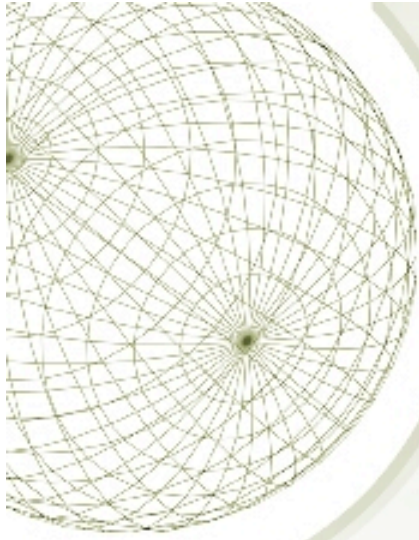
1. Use components

2. Use length and direction



Positive correlation





Negative correlation



$$\vec{V} = (4, -5, 8)$$

$$\vec{W} = (4, -6, 6)$$

$$\alpha \vec{V} + \beta \vec{W} = \vec{0}$$

$$\vec{X} \perp \vec{V} \text{ also } \vec{W}$$

2 ways to multiply \vec{V}, \vec{W}

Dot, or scalar product.

$$\vec{V} \cdot \vec{W}$$

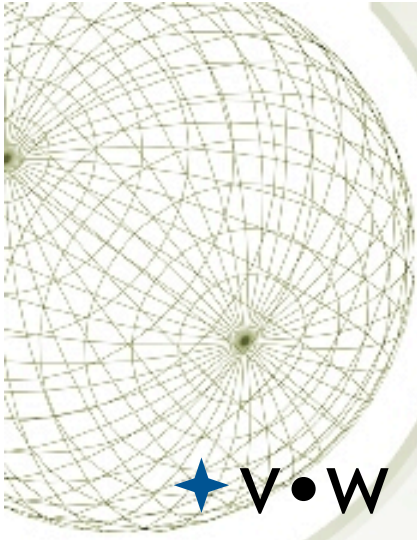
The dot (or scalar) product

Thou shalt know this!

$$\vec{V} \cdot \vec{W} = |\vec{V}| |\vec{W}| \cos \alpha$$

$$= V_1 W_1 + V_2 W_2 + V_3 W_3$$

Vectors in
Scalar out!



Correlation and the dot product

★ $v \cdot w > 0$ Positive correlation

★ $v \cdot w < 0$ Negative correlation

★ $v \cdot w = 0$ Orthogonal

= perpendicular

= uncorrelated



Thou shalt know this!

The cross (or vector) product

★ 2 vectors in, vector out

★ $\mathbf{v} \times \mathbf{w} = |\mathbf{v}| |\mathbf{w}| \sin(\alpha) \mathbf{z}$, where \mathbf{z} is a unit vector perpendicular to both \mathbf{v} and \mathbf{w} , according to the right-hand rule.

★ $|\mathbf{v} \times \mathbf{w}| = \text{area of parallelogram formed from } \mathbf{v} \text{ and } \mathbf{w}.$

★ $\mathbf{v} \times \mathbf{w} = (v_2 w_3 - v_3 w_2) \mathbf{i} + (v_3 w_1 - v_1 w_3) \mathbf{j} + (v_1 w_2 - v_2 w_1) \mathbf{k}$



Some tricky stuff

Cross products. $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. (In fact) What's right is right and what's left is left. Same for calculus.

2 curves

$$\vec{r}_a(t) + \vec{r}_b(t)$$

$$= (x_a(t) + x_b(t), y_a(t) + y_b(t), z_a(t) + z_b(t))$$

$$t \rightarrow x(t)$$

$$\downarrow y(t)$$

$$\downarrow z(t)$$

If I put these together

$$t \rightarrow \vec{r}(t) = (x(t), y(t), z(t))$$
$$= x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$



*One of the great tricks of
vector calculus:*

★ **If you can rewrite a vector
problem in some way as a
scalar problem, it becomes
“kindergarten math.”**

Vectors and Functions

Curve. At a certain time (scalar), just where are you (vector)?

$$t \rightarrow \mathbf{x}(t).$$

Scalar field. At a certain place (vector), just how hot are you (scalar)?

$$\mathbf{x} \rightarrow T(\mathbf{x})$$

Vector field. At a certain place (vector), what are the wind speed and direction (vector)?

$$\mathbf{x} \rightarrow \mathbf{v}(\mathbf{x})$$

Vectors and Functions

Curve. At a certain time (scalar), just where are you (vector)?

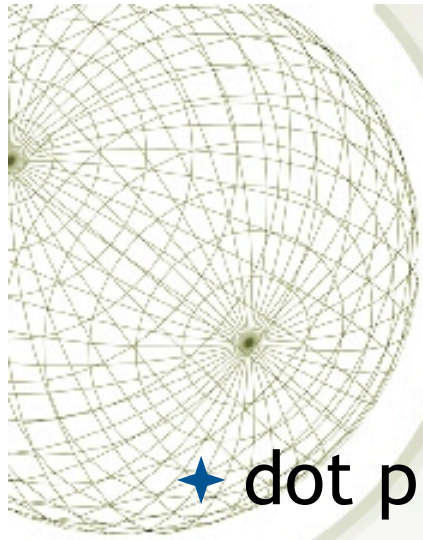
$$t \rightarrow \mathbf{x}(t).$$

Scalar field. At a certain place (vector), just how hot are you (scalar)?

$$\mathbf{x} \rightarrow T(\mathbf{x})$$

Vector field. At a certain place (vector), what are the wind speed and direction (vector)?

$$\mathbf{x} \rightarrow \mathbf{v}(\mathbf{x})$$



Why on earth would you differentiate a

- ★ dot product
- ★ cross product ?



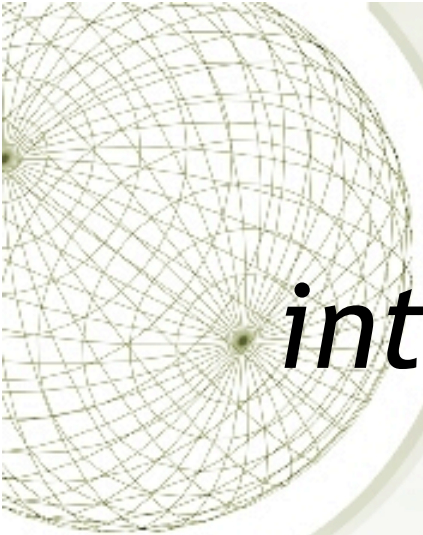
Examples

- ★ How fast is the angle between two vectors changing?

$$\cos \theta(t) = \mathbf{v}(t) \cdot \mathbf{w}(t)$$

(You'll need product and chain rule.)

- ★ How fast is the angular momentum changing? $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

A wireframe sphere is positioned in the upper-left corner of the slide. It is composed of a grid of thin, light-colored lines that form a spherical shape, with a central point where the lines converge.

*Why on earth would you
integrate a vector function?*



Examples

- ★ Given velocity $\mathbf{v}(t)$ find position $\mathbf{x}(t)$.
- ★ Power = $\mathbf{F} \cdot \mathbf{v}$ Work is the integral of this. If, say, \mathbf{v} is fixed, you can integrate \mathbf{F} and then dot it with \mathbf{v} .



The good news:

- ★ The rules of vector calculus look just like the rules of scalar calculus



The good news:

- ★ The rules of vector calculus look just like the rules of scalar calculus
- ★ Integrals and derivs of $\alpha \mathbf{f}(t)$, $\mathbf{f}(t)+\mathbf{g}(t)$, etc.



The good news:

- ★ The rules of vector calculus look just like the rules of scalar calculus
- ★ Integrals and derivs of $\alpha \mathbf{f}(t)$, $\mathbf{f}(t)+\mathbf{g}(t)$, etc.
- ★ Also - because of this - you can always calculate component by component.



Some nice curves

★ 2D spiral:

$$(x(t), y(t)) = (t \cos(t), t \sin(t))$$

★ Lissajoux figure:

$$(x(t), y(t)) = (\cos(2 t), \sin(3 t))$$

★ 3D helix:

$$(x(t), y(t), z(t)) = (4\cos(t), 4\sin(t), t)$$