Mathematics 2411 Final Exam Thursday, 4 May 2000

NAME_____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "**specific expression**" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 | __

2 | __

3 | __

4 | __

5 | __

6 | __

TOTAL _____

NAME_____

1. (10 points) Find the area of the region bounded by the line y=x and the curve

$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} = (t^3, 2t^2),\ 0 \le t \le 2$

a) Express, but do not evaluate, the area as a **specific** line integral:

AREA = _____

b) Express, but do not evaluate, the area as a **specific** integral in the changed variables $u = x$, $v = y^3/x^2$. (Hint: if x=y, then v = u.)

AREA = _____

c) Evaluate the area:

AREA = _____

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

2. Let $\mathbf{F} = x^3\,\mathbf{i} + (x + z)\,\mathbf{j} + (4y - z)\,\mathbf{k}$. and consider the solid Ω bounded by z=2 and the cone $\varphi = \pi/6$. (The angle φ is the colatitude.)

a) Express, but do not evaluate, the integral $\int_{\partial\Omega} \mathbf{F}\ \ \mathbf{n}\ |d^2\mathbf{x}|$
as a **specific** integral:

ANSWER = _____

b) Evaluate the integral:

ANSWER = _____

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

NAME_____

KEY FORMULA OR METHOD (optional for partial credit)_____

3. A topographic map such as the one shown here, of the Chattahoochee National Recreation Area just north of Atlanta, is a contour plot for a function f(x,y). The contours are level sets for values of z=f(x,y). Contours are separated by heights of 10 feet (every fifth contour is printed darker). The horizontal scale is such that the square shown is 2500 feet on a side.

Annotate the contour map as follows:

a) Find the top of a hill and label it with the letter **T**.

b) Find a saddle point and label it with the letter **S**.

c) There are several cliffs in the park. Find a cliff on this map and label it with the letter **C**.

d) In the lower left part of the map you will find a small arrow. Estimate the gradient at the point of the arrow. By the way, the arrow drawn is not meant to indicate the gradient at its base.) Draw a vector on the map with the same direction as the gradient, and estimate the magnitude of the gradient here:_____

KEY FORMULA OR METHOD (optional for partial credit)_____

_____

NAME_____

4. (10 points) The curve C

(x(t), y(t), z(t)) = (2 cos(2t), 2 sin(2t), 4 cos(2t))

is an ellipse.

a) Find the maximum value of f(x,y,z) = x - 2 y + 3 z on C, or explain why it is not defined.

ANSWER: _____

b) Find the unit tangent, normal, and binormal vectors to C at time t :

t(t) = _____

n(t) = _____

b(t) = _____

c) Find the formula for the plane containing C:

ANSWER: _____

d) Find the maximal value of the curvature of the ellipse:

The maximal value of the curvature is _____, and it occurs at _____.

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

5. (10 points) Evaluate

$$\int (3x^2y\ dx + x^3dy + 2\ dz)$$

a) over the curve given in problem 4 for $0 \le t \le \pi/2$:

ANSWER:_____

b) over the curve given in problem 4 for $0 \le t \le 2\pi$:

ANSWER:_____

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

6. (10 points) The surface Ω defined by

$$5x^2 + y^2 + z^2 - 4x\,z = 4,\ z > 2x$$

is half of an ellipsoid. (Possibly helpful: $x^2 + y^2 + (z-2x)^2 = 4$.) Let **n** be the upward normal vector to the ellipsoid and define the vector field:

$$\mathbf{F} = (3\,y + 3\,x^2 y)\mathbf{i} + (x + x^3)\,\mathbf{j} + 2\,\mathbf{k}\ .$$

Evaluate:

$$\iint_{\partial\Omega} \times \mathbf{F}\ \ \mathbf{n}\left|d^2\mathbf{x}\right| = \_\_\_\_\_$$

KEY FORMULA OR METHOD (optional for partial credit)_____