Mathematics 2411 Test number 1 Friday, 28 January 2000

NAME____________________________

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 |__

2 |__

3 |__

4 |__

TOTAL________

NAME____________________

In the following problems we consider the parametric curve C given by

$x(\tau) = \cos^3(\pi\tau)$, $y(\tau) = \sin^3(\pi\tau)$

and a function of two variables $F(x,y) = \sin(\pi x) \cos(2 \pi y)$.

1 (10 points)

a) Find the speed, velocity, and acceleration of the point $\mathbf{r}(\tau)$.

The speed = ____________________________

$\mathbf{v}(\tau)$ = ____________________________

$\mathbf{a}(\tau)$ = ____________________________

b) The curve C touches the positive x-axis and the positive y-axis. Find the arclength of the curve between those intercepts.

The arclength = ____________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________

2 (10 points) Find the unit tangent vector, the unit normal vector, and the curvature of the curve C at parameter value τ.

$\mathbf{t}$ = __________________________

$\mathbf{n}$ = __________________________

k = __________________________

KEY FORMULA OR METHOD (optional for partial credit)____________________

NAME____________________

3 (10 points)

Find the angle bewteen the curve C and the line from the origin which hits the curve where $\tau = 1/3$. (Numerical value of answer is OK).

The angle = ____________________________

4 (10 points) For F(x,y) as on the previous page, find:

a) $\frac{\partial F}{\partial x}\left(\frac{1}{3},\frac{2}{3}\right)$ = ______________________________

b) The directional derivative of F at (1/3,2/3) in the direction of $\mathbf{v} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

ANSWER:__________________________________

c) All points where the grad F = **0**.

ANSWER:__________________________________

KEY FORMULA OR METHOD (optional for partial credit)___________________

EXTRA CREDIT PROBLEMS, IN CASE YOU HAVE TIME ON YOUR HANDS

ALTERNATIVELY, YOU CAN DO THESE FOR A CONTEST, DUE NEXT FRIDAY

EC1: Where is the center of the smallest circle which osculates C in the first quadrant?

EC2: Find the velocity and acceleration of **r**(τ) on the curve C, as expressed in the basis **t**, **n**

EC3: Find the arclength of the space curve

$$x(\tau) = \cos^{3}(\pi\tau),\ y(\tau) = \sin^{3}(\pi\tau),\ z(\tau) = \sin^{2}(\pi t)$$

from the x-z plane to the y-z plane

EC4: Find the vectors **t**, **n**, and **b**, as well as the curvature and the torsion for the space curve of EC2.

EC5: Discuss: If I know the curvature of a curve k(s), as a function of arc length, to what extent is the curve determined? (For a space curve, you may wish to consider $\tau(s)$ as well.)