NAME: Pink solns

3. Consider the curve $\mathbf{r}(t) = \cos(\pi t^2)\mathbf{i} - t^2\mathbf{j} + \sin(\pi t^2)\mathbf{k}$. Calculate

a) The arclength from $t = 0$ to $t = 2\pi$.

ANSWER $\int_0^{2\pi} 2t\sqrt{1+\pi^2}\,dt = \boxed{\sqrt{1+\pi^2}\cdot 4\pi^2}$

b) The unit tangent vector at time t:

ANSWER $\frac{1}{\sqrt{1+\pi^2}}\left(-\pi\sin(\pi t^2)\hat{i} - \hat{j} + \pi\cos(\pi t^2)\hat{k}\right)$ $\quad \vec{r}'/\|\vec{r}'\|$

c) The curvature $\kappa(t) =$ $\boxed{\pi^2/(1+\pi^2)}$

$\dot{r} = -2\pi t\sin(\pi t^2)\hat{i} - 2t\hat{j} + 2\pi t\cos(\pi t^2)\hat{j}$

$\|\dot{r}\| = t\sqrt{4+4\pi^2} = 2t\sqrt{1+\pi^2}$

$K = \left\|\frac{d\vec{T}}{ds}\right\| = \left(\frac{1}{ds/dt}\right)\left\|\frac{d\vec{T}}{dt}\right\| = \frac{(-4\pi^2\cos(\pi t^2)\hat{i} - 4\pi^2\sin(\pi t^2)\hat{k})}{2t\sqrt{1+\pi^2}\sqrt{4\pi^2}}$

4. Suppose that a particle moves along the helical path given in Problem 3. Find

a) The angle between the particle's path and the line

$$\mathbf{r}_\ell(u) = -u\mathbf{i} - u\mathbf{j} + (1-u)\mathbf{k}$$

at the point $(-1,-1,0)$ where they intersect.

$\arccos\left(\frac{\sqrt{2}}{\sqrt{4+4\pi^2}}\right) =$

They intersect when $u = t = 1$

$\vec{r}'(1) = -2\pi\sin(\pi)\hat{i} - 2\hat{j} + 2\pi\cos(\pi)\hat{k}$

$\vec{r}_\ell'(1) = -\hat{i} - \hat{j}$

$\therefore \frac{\vec{r}_\ell'(1)\cdot\vec{r}'(1)}{\|\vec{r}_\ell'(1)\|\,\|\vec{r}'(1)\|} = \frac{2}{\sqrt{2}\sqrt{4+4\pi^2}}$

b) The acceleration in the local frame

$$\frac{d^2\mathbf{r}}{dt^2}(t) = \boxed{2\sqrt{1+\pi^2}\ \mathbf{T} + 4\pi^2 t^2\ \mathbf{N} + 0\ \mathbf{B}}$$

EC.) Optional for extra credit (2 points). Calculate

$$\frac{d\mathbf{N}}{dt}(t) = \quad \mathbf{T} \quad \mathbf{N} \quad \mathbf{B}$$

Use $\vec{r}' = \frac{ds}{dt}\hat{T} = 2t\sqrt{1+\pi^2}\,\hat{T}$

$\vec{r}'' = \ddot{s}\hat{T} + K\dot{s}^2\hat{N} = 2\sqrt{1+\pi^2}\,\hat{T} + \frac{\pi^2}{1+\pi^2}\cdot 4t^2(1+\pi^2)\,\hat{N}$