Test 1 solutions (pink version)

1 (10 points). Calculate the following:

a)

```
> vec := t -> crossprod([0, 3*t ,6*t ], [exp(t), t ,2*t ]);
```

$$vec := t \to crossprod\left([0, 3\,t, 6\,t], \left[e^t, t, 2\,t\right]\right) \quad (1)$$

```
> map(diff,vec(t), t);
```

$$\left[\begin{array}{ccc} 0 & 6\,e^t + 6\,t\,e^t & -3\,e^t - 3\,t\,e^t \end{array}\right] \quad (2)$$

b)

```
> map(int,[cos(t/2),t/(t^2-1)], t=2..5);
```

$$\left[2\sin\left(\frac{5}{2}\right) - 2\sin(1), \frac{3}{2}\ln(2)\right] \quad (3)$$

This is the best frm of the answer, but in case you prefer numerical approximations:

```
> evalf(%);
```

$$[-.485997682, 1.039720771] \quad (4)$$

2 (10 points). On the curve shown below, clearly indicate

a) the unit tangent vector at the point P. Label it T

b) the principal unit normal vector at the point P. Label it N

c) at least one point where the curvature is maximized. Label it M.

For part a), the unit tangent vector can point in either direction. For part b), the unit normal vector should point to the inside of the curve and have the same length as T. For part c, M should be where the curve is sharpest.

In problems 3 and 4, an object of mass 2 moves along the helix

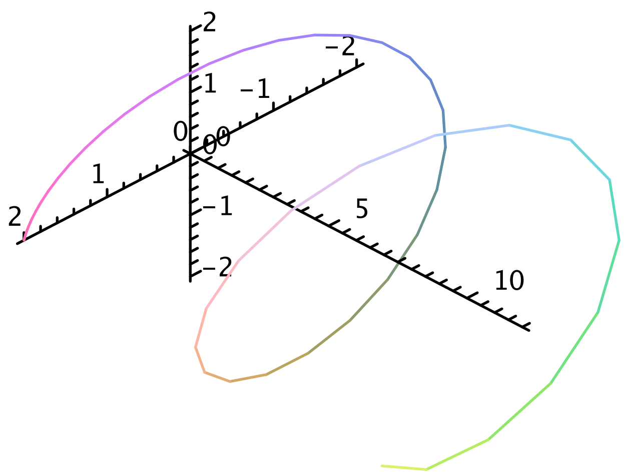
r(t) = 2 cos(3 t^2) i + 3 t^2 j + 2 sin(3 t^2) k

```
> r := t -> vector([2*cos(3*t^2),3*t^2,2*sin(3*t^2)]);
```

$$r := t \to \left[\begin{array}{ccc} 2\cos\left(3\,t^2\right) & 3\,t^2 & 2\sin\left(3\,t^2\right) \end{array}\right] \quad (5)$$

First, let's look at it:

```
> with(plots): spacecurve([2*cos(3*t^2),3*t^2,2*sin(3*t^2)], t=0..2, axes=NORMAL);
Warning, the name changecoords has been redefined
```



3 (10 points).

a) The speed of the object at t= 3 is

```
> with(linalg): map(diff,r(t),t): norm(%, 2);
Warning, the protected names norm and trace have been redefined and
unprotected
```

$$6\sqrt{4\left|\sin\left(3\,t^2\right)\,t\right|^2+\left|t\right|^2+4\left|\cos\left(3\,t^2\right)\,t\right|^2} \quad \textbf{(6)}$$

which simplifies to 6 t sqrt(5). At t=3, it is **18sqrt(5)**.

b) Tthe length of the curve between the points (2,0,0) and (-2,3 π,0) is the same as the curve from t=0 to t=sqrt(π):

```
> int(6*t*sqrt(5), t=0..sqrt(Pi));
```

$$3\sqrt{5}\,\pi \quad \textbf{(7)}$$

c) As for the tangent line at t=sqrt(Pi)/3, it passes through

```
> r(sqrt(Pi)/3);
```

$$\left[\begin{array}{ccc} 1 & \frac{1}{3}\pi & \sqrt{3} \end{array}\right] \quad \textbf{(8)}$$

and has direction given by the velocity

```
> map(diff,r(t),t);
```

$$\left[\begin{array}{ccc} -12\sin\left(3\,t^2\right)\,t & 6\,t & 12\cos\left(3\,t^2\right)\,t \end{array}\right] \quad \textbf{(9)}$$

at t= sqrt(π)/3:

```
> subs(t=sqrt(Pi)/3, %);
```

(10)

$$\left[\ -4\sin\left(\frac{1}{3}\pi\right)\sqrt{\pi}\quad 2\sqrt{\pi}\quad 4\cos\left(\frac{1}{3}\pi\right)\sqrt{\pi}\ \right] \tag{10}$$

```
> simplify(%);
```

$$\left[\ -2\sqrt{3}\sqrt{\pi}\quad 2\sqrt{\pi}\quad 2\sqrt{\pi}\ \right] \tag{11}$$

```
>
```

Hence the line is

R(u) = (1 - 2 sqrt(3 Pi) u) i + (Pi/3 + 2 sqrt(Pi) u) j + (sqrt(3) + 2 sqrt(Pi) u) k.

4.

a) Find the force on the object in the moving coordinate system.

There is no **B** component, because the force is just 2* acceleration, which is

s" **T** + (s')^2 k **N**.

hence we need to calculate the speed, its derivative, and the curvature.

We already know the speed,

```
> speed := t -> 6*sqrt(5)*t;
```

$$speed := t \rightarrow 6\sqrt{5}\,t \tag{12}$$

so the tangential acceleration is 6 sqrt(5). The only tricky part is the curvature, but since the curve is a helix, it won't be too bad.

```
> crossprod(map(diff,r(t),t), map(diff,r(t),t$2));
```

$$\left[6t\left(-72\sin(3t^2)t^2+12\cos(3t^2)\right)-72\cos(3t^2)t, 12\cos(3t^2)t\left(-72\cos(3t^2)t^2 - 12\sin(3t^2)\right)+12\sin(3t^2)t\left(-72\sin(3t^2)t^2+12\cos(3t^2)\right), -72\sin(3t^2)t - 6t\left(-72\cos(3t^2)t^2-12\sin(3t^2)\right)\right] \tag{13}$$

```
> simplify(norm(%,2)/(6 * sqrt(5)* t)^3);
```

$$\frac{2}{25}\,\frac{\sqrt{\left|\left(-1+\cos(3t^2)^2\right)t^6\right|+4|t|^6+\left|\cos(3t^2)^2 t^6\right|}\,\sqrt{5}}{t^3} \tag{14}$$

Come on, Maple, can't you see the common factor of t^3 in the numerator and denominator? And what about the cancellation of the cosines? Sheesh!

```
> curvature := 2/5;
```

$$curvature := \frac{2}{5} \tag{15}$$

The **T** component of the force is **12 sqrt(5)** and the **N** component is

```
> 2* speed(t)^2 * curvature;
```

$$144\,t^2 \tag{16}$$

```
>
```