

NAME _____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 | _____

2 | _____

3 | _____

4 | _____

TOTAL _____

NAME _____

1 (10 points) Let S denote the surface defined by

$$x y z = 4$$

a) Find a normal vector at the point (1,2,2):

$$\mathbf{N} =$$

b) Find a basis for the tangent space at (1,2,2). (That is, find two independent vectors which are tangential to S at that point.)

$$\mathbf{v}_1 =$$

$$\mathbf{v}_2 =$$

c) Give the formula for the tangent plane to S at (1,2,2):

$$\text{_____} = \text{_____}$$

KEY FORMULA OR METHOD (optional for partial credit) _____

2 (10 points) Let S denote the surface defined in problem 1. Find the point nearest the origin

XXX

$$x y z = 4$$

a) Find a normal vector at the point (1,2,2):

$$\mathbf{N} =$$

b) Find a basis for the tangent space at (1,2,2). (That is, find two independent vectors which are tangential to S at that point.)

$\mathbf{v}_1 =$

$\mathbf{v}_2 =$

c) Give the formula for the tangent plane to S at (1,2,2):

_____ = _____

KEY FORMULA OR METHOD (optional for partial credit)_____

NAME _____

3 (10 points)

Find the angle between the curve C and the line from the origin which hits the curve where $\tau = 1/3$. (Numerical value of answer is OK).

The angle = _____

4 (10 points) For $F(x,y)$ as on the previous page, find:

a) $\frac{\partial F}{\partial x} \left(\frac{1}{3}, \frac{2}{3} \right) =$ _____

b) The directional derivative of F at $(1/3, 2/3)$ in the direction of $\mathbf{v} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

ANSWER: _____

c) All points where the grad $F = \mathbf{0}$.

ANSWER: _____

KEY FORMULA OR METHOD (optional for partial credit) _____

EXTRA CREDIT PROBLEMS, IN CASE YOU HAVE TIME ON YOUR HANDS

ALTERNATIVELY, YOU CAN DO THESE FOR A CONTEST, DUE NEXT FRIDAY

EC1: Where is the center of the smallest circle which osculates C in the first quadrant?

EC2: Find the velocity and acceleration of $\mathbf{r}(\tau)$ on the curve C , as expressed in the basis \mathbf{t}, \mathbf{n}

EC3: Find the arclength of the space curve

$x(\tau) = \cos^3(\pi\tau), y(\tau) = \sin^3(\pi\tau), z(\tau) = \sin^2(\pi\tau)$
from the x - z plane to the y - z plane

EC4: Find the vectors \mathbf{t}, \mathbf{n} , and \mathbf{b} , as well as the curvature and the torsion for the space curve of EC2.

EC5: Discuss: If I know the curvature of a curve $k(s)$, as a function of arc length, to what extent is the curve determined? (For a space curve, you may wish to consider $\tau(s)$ as well.)