

NAME _____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. An "explicit expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 | _____

2 | _____

3 | _____

TOTAL _____

NAME _____

1 (10 points) (This problem is a clone of H², 4.5.16.) Find the volume of the region bounded by the surfaces

$$z = x^2 + y^2$$

and

$$z = 4 - 15x^2 - 15y^2$$

Structure your answer as follows:

a) Write, but do not evaluate, an explicit double integral for this volume in the Cartesian system by filling in the blanks here:

$$Volume = \int_{\underline{\quad}}^{\underline{\quad}} \int_{\underline{\quad}}^{\underline{\quad}} \underline{\quad} d\underline{\quad} d\underline{\quad}.$$

b) Write, but do not evaluate, an explicit double integral for this volume in a non-Cartesian system by filling in the blanks here:

$$Volume = \int_{\underline{\quad}}^{\underline{\quad}} \int_{\underline{\quad}}^{\underline{\quad}} \underline{\quad} d\underline{\quad} d\underline{\quad}.$$

c) Evaluate the volume of the region.

$$Volume = \underline{\quad}$$

d) Write the second set of coordinates in terms of the Cartesian coordinates, and calculate the Jacobian in terms of the second set:

$$x = \underline{\quad}, \quad y = \underline{\quad}$$

$$J = \underline{\quad}$$

(should be a function of the "second set" of variables)

e) Write the Cartesian coordinates in terms of the second set of coordinates, and calculate the Jacobian in terms of x and y:

$$\underline{\quad} = \underline{\quad}, \quad \underline{\quad} = \underline{\quad}$$

$$J = \underline{\quad}$$

(should be a function of x,y)

KEY FORMULA OR METHOD (optional for partial credit) _____

