Mathematics 2401N **Test 4** **19 November 2007 VERSION** Υ

NAME: _____

TA: _____

Instructions: Work absolutely on your own, without reference to notes or text. Answers should be as specific as possible and it should be evident how they were obtained. Write the answers where indicated.

This test will end promptly at 9:55. Sign below and await the signal to begin the test.

IMPORTANT. In this test your grade will be based on the best three of four answers. You may concentrate on your choice of three problems, or you may choose to attempt four, in which case the problem on which you receive the lowest score will not be counted.

I am familiar with the Georgia Tech Honor Code and will abide by it. Any stored information about MATH 2401 has been erased from my calculator (or similar storage device)

_____

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1. (10 points) In this problem the path of integration C is the upper half of the circle of radius 2, traversed counterclockwise, followed by the straight line segment from (-2,0) to (2,0). We consider two vector fields that differ only slightly,
$\mathbf{F}_1(x,y) = 3y\mathbf{i} + (3x+2)\mathbf{j}$, and $\mathbf{F}_2(x,y) = 4y\mathbf{i} + (3x+2)\mathbf{j}$.

   a) Write the curve in parametric form (2 pieces OK):

   _____

   b) Is $\mathbf{F}_1$ the gradient of a scalar function? (_____Y _____N) If so, $\mathbf{F}_1 = \nabla$_____.

   c) Is $\mathbf{F}_2$ the gradient of a scalar function? (_____Y _____N) If so, $\mathbf{F}_2 = \nabla$_____.

   d) Evaluate:

$$\int_C (3ydx + (3x+2)dy) = \_\_\_\_\_$$

   e) Evaluate:

$$\int_C \mathbf{F}_2 \cdot d\mathbf{r} = \_\_\_\_\_$$

2. (10 points) Take Ω as the parallelogram bounded by

$$x - y = 0, x - y = 2, x + 2y = 0, x + 2y = 4$$

   Evaluate

$$\int_\Omega \int y dx dy.$$

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3. (10 points) Find the mass of the region Υ formed by the intersection of the wedge $0 \leq y \leq x/\sqrt{3}$ with the ball of radius 10 cm centered at the origin. (Υ is shaped like an orange slice with wedge angle $\frac{\pi}{6}$.) Assume that the density is $\frac{|\mathbf{r}|+5}{|\mathbf{r}|}$, where $\mathbf{r}$ designates the position vector.

   A SPECIFIC INTEGRAL REPRESENTING THE MASS IS:

   _____

   THE MASS EQUALS: _____

4. (10 points) The region Ω lies above the xy-plane, below the hemisphere $x^2+y^2+z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$.

   a) Write a specific integral for the volume of Ω:

   _____

   b)

   $Vol(\Omega) =$ _____

   c) Find the centroid of Ω:

   $x_M =$ _____

   $y_M =$ _____

   $z_M =$ _____