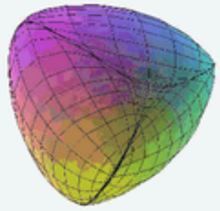


Prof. Harrell's Math 2411
Georgia Tech, Spring, 2013

School of Mathematics
Skiles Building, Office 218D
Georgia Institute of Technology
Atlanta GA 30332-0160

email: *harrell at math.gatech.edu*
Phone: (404) 894 3300

Current items of interest

- The syllabus
- An example of mathematical literature
- Some entertaining videos
  1. Mathemagic
  2. How to turn a sphere inside out

Useful resources

- The class syllabus
- The vector calculus page at mathphysics.com
- On-line lectures: *Calculus* by George Cain and James Herod.
- The bank of old tests
- History of Mathematics
- Mathworld (reference for mathematical theorems and facts)
- Treasure Troves of Science
- Maple

MATH 2411 - Harrell

Visualization in 3D

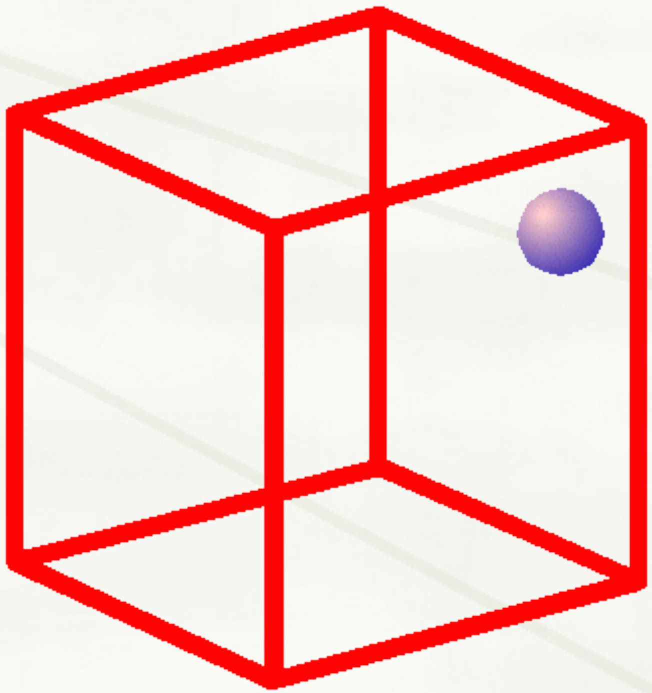
Lecture 1

This week's learning plan

- You will begin to think about visualizing in 3D and the math thereof.
- You will review the basic vector operations and make sure you have mastered them.
- You will start differentiating vectors and know why.

Visualization in 3D

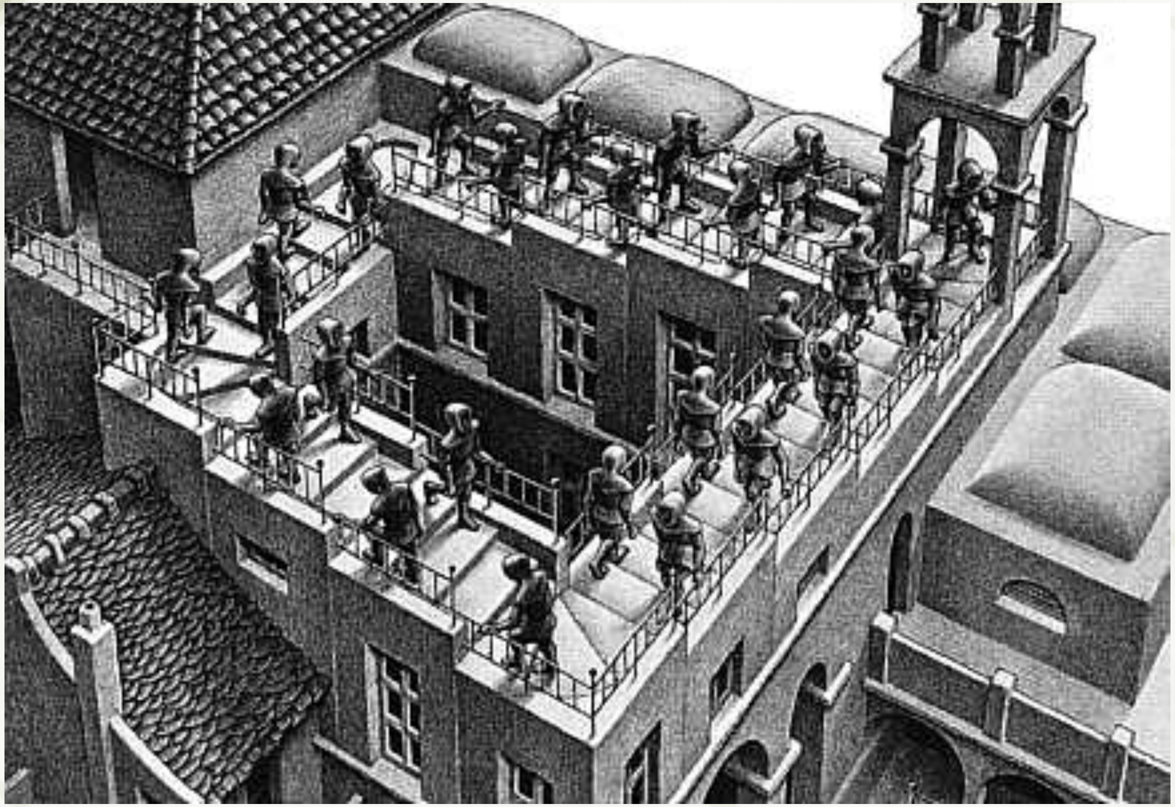
How well can you visualize? Is the ball on a front or rear face of the cube?



Can you visualize this 4 different ways?

Visualization in 3D

How well can you visualize? Which of these pictures are possible?



M.C. Escher

Visualization in 3D

How well can you visualize? Which of these pictures are possible?

Borromean rings

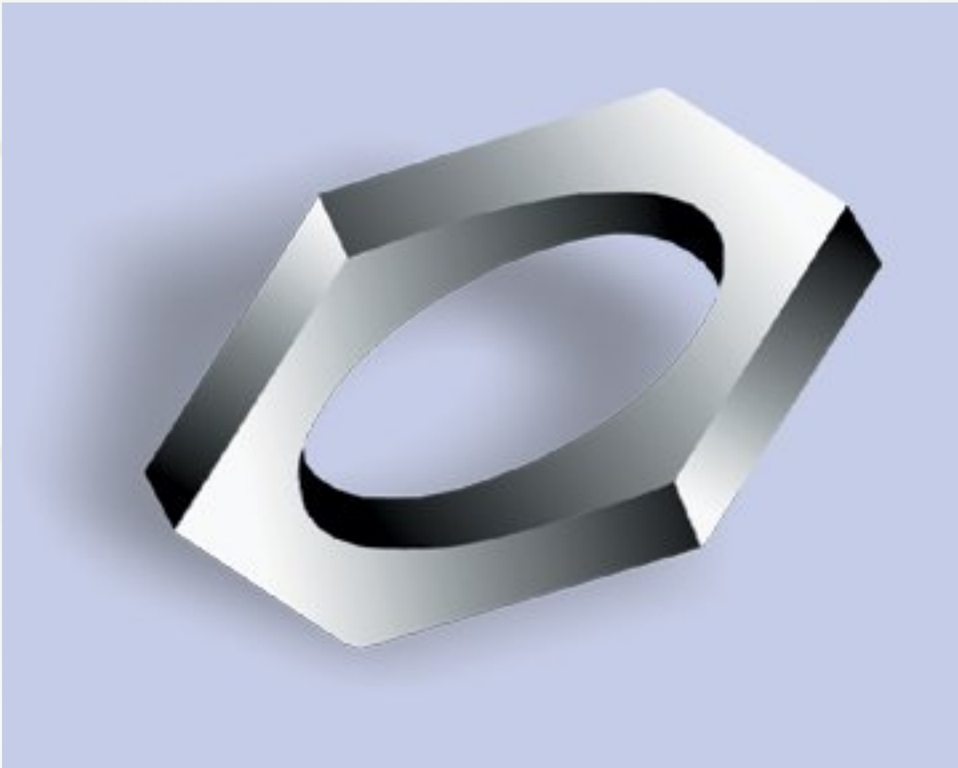
Visualization in 3D

How well can you visualize? Which of these pictures are possible?

This was not invented by Mad Magazine!

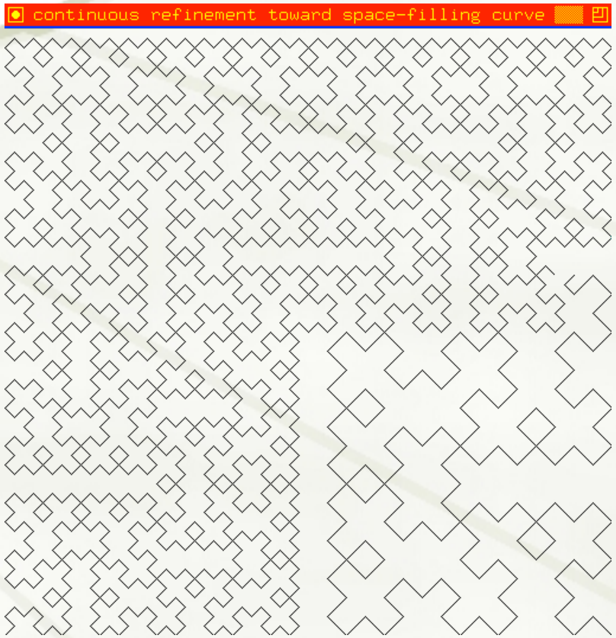
Visualization in 3D

How well can you visualize? Which of these pictures are possible?



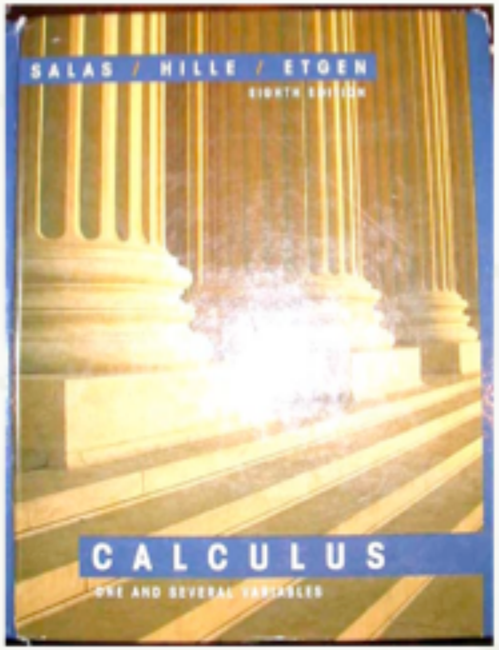
Visualization in 3D

How well can you visualize? Which of these pictures are possible?

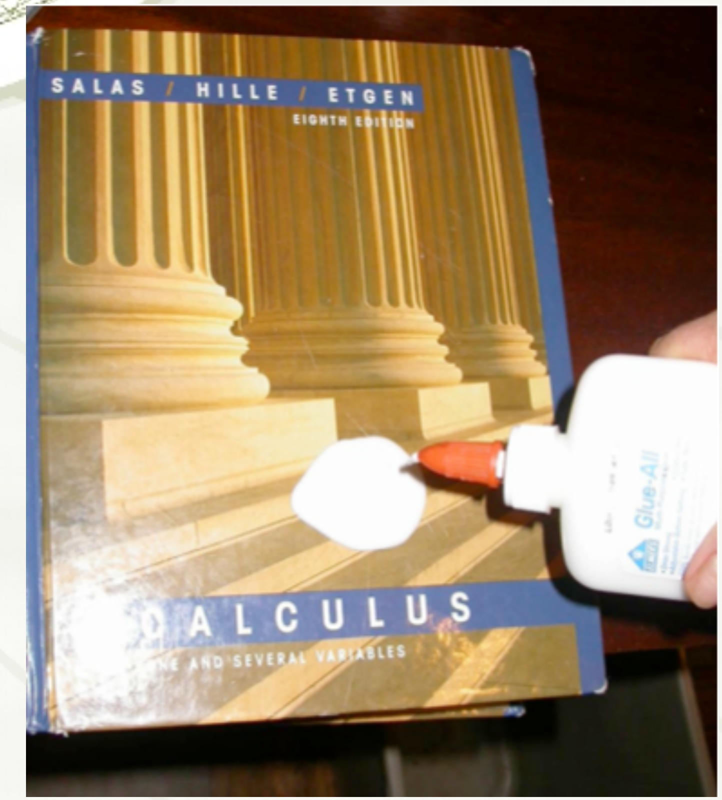


Can a continuous curve pass through every point of space?

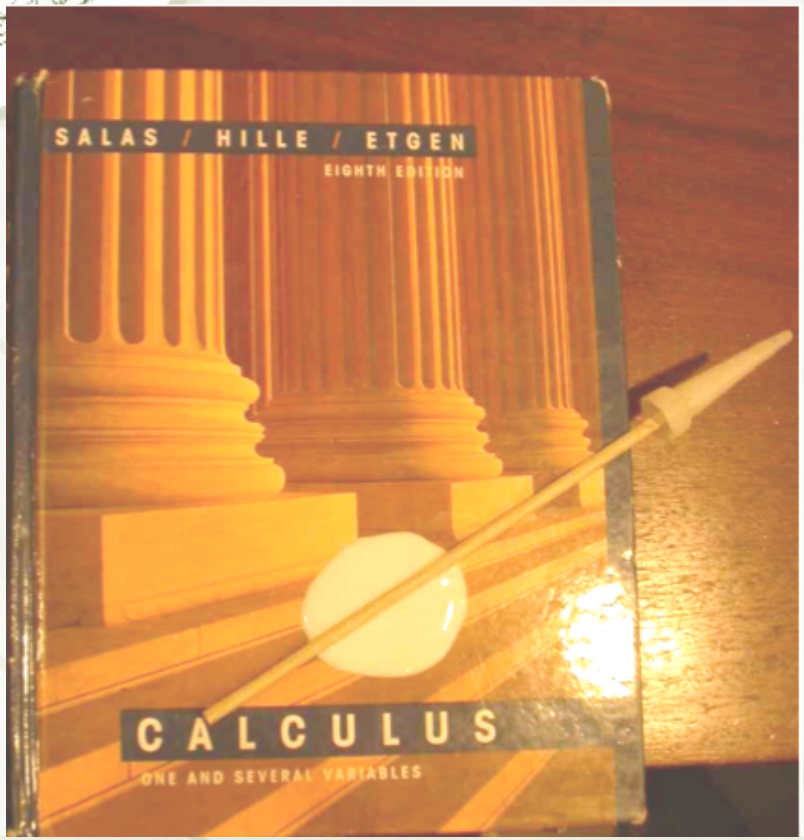
Just to see what happens, let’s glue together two ideas



Just to see what happens, let's glue together two ideas



Just to see what happens, let's glue together two ideas



Ta-da!


But first - A bit more Vector Boot Camp!
VASQUE
VASQUE

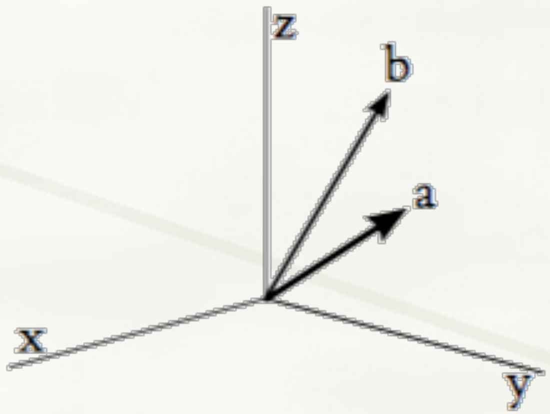
Two ways to think about vectors

1. As a list of components

2. Something with length and direction

REVIEWARAMA

Figure 1:



1) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown in Figure 1. The vector $\mathbf{a}$ lies in the first octant with all components positive. The vector $\mathbf{b}$ is in the second octant, i.e. $\mathbf{b} \cdot \mathbf{i} < 0$ but the y and z components are positive. Label the six figures below as $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $\mathbf{b}-\mathbf{a}$, $\mathbf{a}\times\mathbf{b}$, $\mathbf{b}\times\mathbf{a}$, $\mathbf{a}\times\mathbf{a}$.

3) Find the component of $\mathbf{x}_1$ in the $\mathbf{x}_2$ direction and give a projection matrix for the span of $\mathbf{x}_1$ and $\mathbf{x}_2$.
4) Suppose $\mathbf{a} = (1, 1, 0)$, $\mathbf{b} = (-1, 2, 0)$, and $\mathbf{c} = (1, 2, 3)$. What is the area of the parallelogram with $\mathbf{a}$ and $\mathbf{b}$ as sides and volume of the parallelepiped with $\mathbf{a}, \mathbf{b}, \mathbf{c}$ as sides?
5) Give an equation in variables x, y, z describing the plane containing the points $(0, 1, 1)$, $(1, 0, 1)$, and $-(1, 1, 0)$.
6) Recall the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ for any angle θ. Show that for any vectors $\mathbf{a}$, $\mathbf{b}$ in $\mathbb{R}^3$ the following identity holds:

$$\|\mathbf{a}\|^2\|\mathbf{b}\|^2 = \|\mathbf{a}\times\mathbf{b}\|^2 + (\mathbf{a}\cdot\mathbf{b})^2$$

Clicker quiz

Is the angle between the two vectors

$2\,\mathbf{i} - 3\,\mathbf{j} + 5\,\mathbf{k}$ and $3\,\mathbf{i} - \mathbf{k}$

A acute (angle < $\pi/2$)

B obtuse (angle > $\pi/2$)

C a right angle

The dot (or scalar) product

Thou shalt know this!

The dot (or scalar) product

Thou shalt know this!

- Vectors in..... scalar out.
- Also allows us to define *length*, a.k.a. *norm*, *magnitude*
- The standard length is not the only possibility

The cross product

Thou shalt know this!

The cross product

Thou shalt know this!

- Vectors in..... vector out.

Thou shalt know this!

The product of a scalar and a vector

- Vector and scalar in..... vector out.

An idle question

Are there vectors hiding in the grading formula?

Class web page

The class will be coordinated through T-Square, but you can also consult the Class web page directly. It is your responsibility to consult T-Square or the web page regularly for information about the class, such as homework assignments. You will also be in e-mail contact with the instructor and the teaching assistant, and we will do our best to respond to your questions.

Required texts and materials

We will use Marsden and Tromba, *Vector Calculus*, and on-line materials, which may be linked to from the class website or T-Square. You are also required to have a clicker for classroom use.

Description: Calculus is not only essential in engineering; it is one of mankind's greatest intellectual achievements. After thousands of years of confusion on the part of philosophers, Newton, Leibniz, and Euler created effective conceptual tools for understanding the infinite and the infinitesimal. In the third term of calculus we learn about derivatives and integrals in three (or even more) dimensions and their uses.

Grading and requirements

There will be tests in the recitations on

1. **Wednesday, 23 January**,
2. **Wednesday, 13 February**,
3. **Wednesday, 13 March**,
4. **Wednesday, 10 April**,

There will also be a final exam, of course. Homework will be collected on Mondays, and there will be regular clicker quizzes. Your homework-quiz average will incorporate at least two drops. In addition, Prof. Harrell may announce occasional contests, and the winner of a contest will receive a small number of extra-credit points.

Students' grades will depend on the following quantity:

```
(T1  + T2  + T3  + T4 + HQ  + F  - min(T1..T4,F) )    + E + F/2
```

where the components of this formula correspond to the ingredients mentioned above, after scaling so that all of them except E = extra credit total have a common median of 70. The drop in the formula is *the* mechanism for coping with personal events such as illness and family emergencies. **There will be no opportunities for make-up tests after the fact. In the event of an absence due to travel representing Georgia Tech, such as an intercollegiate sports competition, you must** notify the professor **at least two weeks in advance** to arrange an early test or other alternative. Otherwise, such absences will be treated as personal.

Calculators and tests

No restrictions will be placed on the use of calculators that do *elementary* mathematics on the tests. **Calculators that can do calculus symbolically shall not be brought to tests.** No credit will be given on tests for a correct answer without the intermediate steps.

Readings

The schedule of reading will be posted on the 2411 assignments page and on T-Square. The subject matter covered will be roughly the following:

Parametrized paths, velocity and acceleration, curvature and arclength
Continuity, etc. for functions of several variables. Graphs and contour plots
Partial derivatives and the chain rule. Taylor's theorem in several variables
The gradient, tangent planes, best linear approximation and best quadratic approximation
Optimization, types of critical points
Lagrange multipliers, Newton's method in several variables
Double integrals, iterated integrals. Applications: area, center of mass, and volume by Pappus's Theorem
Area integrals in polar coordinates, general coordinates and the Jacobian in the plane
Triple integrals, spherical and cylindrical coordinates
Jacobian in three variables. Improper multiple integrals and applications
Vector fields and line integrals, work and flux
Green's theorem in the plane for both work and flux integrals. Exact vector fields
Parameterized surfaces and surface integrals
Gauss's Theorem and Stokes's Theorem
Applications

Why on earth would you want to differentiate a

- dot product?
- cross product?

Applications

- How fast is the angle between two unit vectors changing?

$$\cos\theta(t) = \mathbf{v}(t)\cdot\mathbf{w}(t)$$

- What is the rate of change of angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$?

Examples

- How fast is the angle between two unit vectors changing?

$$\cos\theta(t) = \mathbf{v}(t) \cdot \mathbf{w}(t)$$

Examples

- How fast is the angle between two unit vectors changing?

$$\cos\theta(t) = \mathbf{v}(t)\cdot\mathbf{w}(t)$$

$\vec{m}\ (t, t)\quad \vec{n} = (t, 5)$

$\cos\angle(m, n) = \dfrac{\vec{m}\cdot\vec{n}}{\sqrt{2t^2}\sqrt{(t^2+25)}}$

$= \dfrac{5t + t^2}{\sqrt{2t^2(t^2+25)}}$

Examples

- What is the rate of change of angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$?

Why on earth would you want to integrate a vector function?

Why on earth would you want to integrate a vector function?

- Given velocity **v**(t), find position **r**(t).

- Power is **F•v**. If, for example one of these vectors is fixed, can you integrate the other one and *then* take the dot product?

The good news

- The rules of vector calculus look just like the rules of scalar calculus!

The good news

- The rules of vector calculus look just like the rules of scalar calculus!
- Integrals and derivs of α **f(t)**, **f(t)+g(t)**, etc.
- Also - because of this - you can always calculate component by component.

Calculus is built on the idea of a limit. What does a limit mean for vector functions?

The limit

$$\lim_{t \to t_0} \mathbf{f}(t) = \mathbf{L}$$

means

$$\lim_{t \to t_0} \|\mathbf{f}(t) - \mathbf{L}\| = 0$$

Calculus is built on the idea of a limit. What does a limit mean for vector functions?

The limit

$$\lim_{t \to t_0} \mathbf{f}(t) = \mathbf{L}$$

means

$$\lim_{t \to t_0} \|\mathbf{f}(t) - \mathbf{L}\| = 0$$

This may contain vectors, but in the end it is some kind of scalar that depends on t.

One of the great tricks of vector calculus:

- If you can rewrite a vector problem in some way as a scalar problem, it becomes “kindergarten math.”

$$\|\mathbf{f}(t) - \mathbf{L}\|^2 = (f_1 - L_1)^2 + (f_2 - L_2)^2 + (f_3 - L_3)^2$$

- So if the left side $\rightarrow 0$, each and every one of the contributions on the right $\rightarrow 0$ as well. And conversely.
- *You can do calculus in terms of vectors or components. Your choice.*

You can think in terms of a holistic vector or its components

A mystical picture: