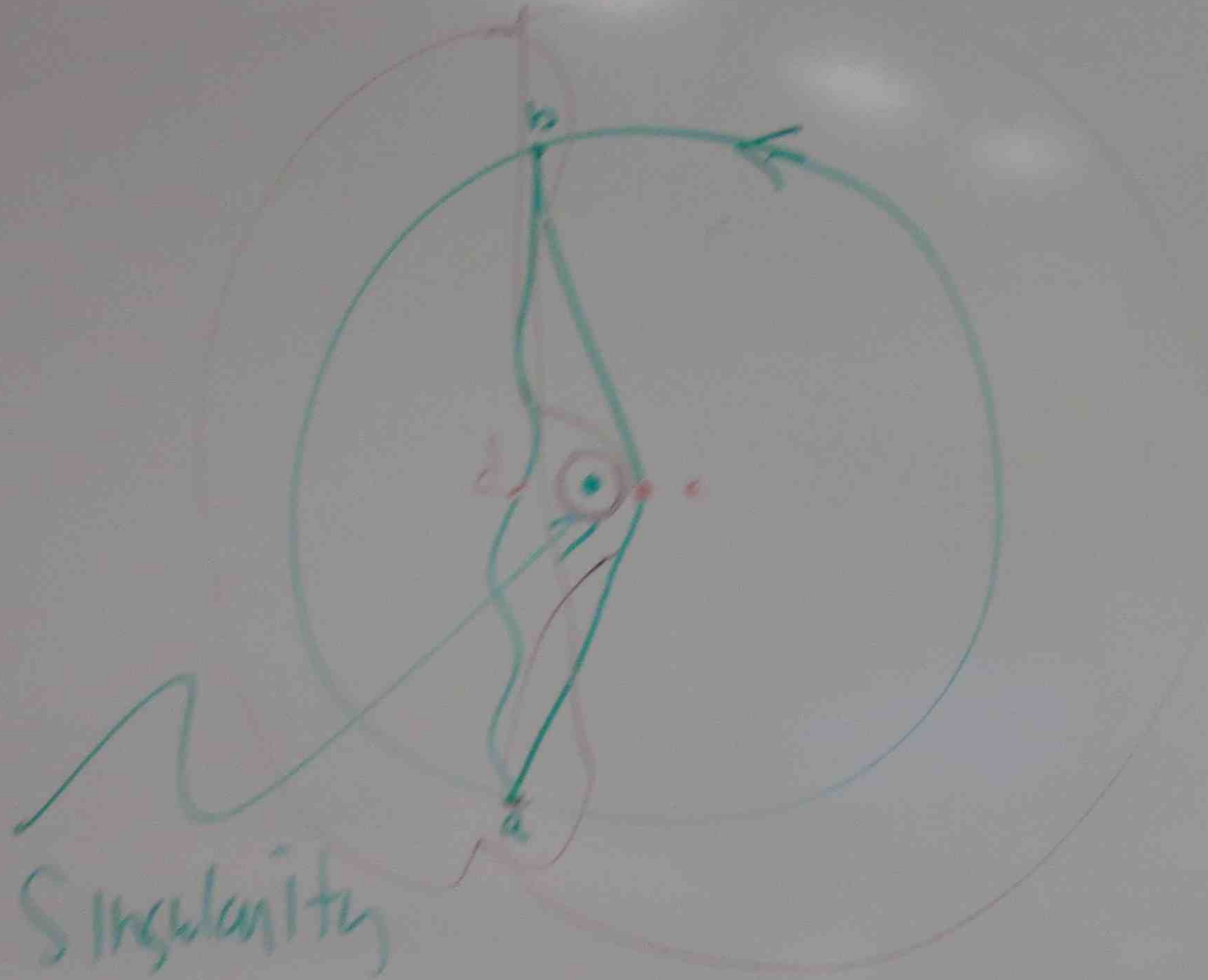


If

$\vec{V} = \nabla f$

Then the integral is
indep. of path.
within a region
that is simply connected.

test fn is $\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}$

By path independence,

$$\to \oint_{big} \bar{V} \cdot d\bar{s} = \oint_{little} \bar{V} \cdot d\bar{s}$$

M

$$\oint \frac{-y^3}{(x^2+y^2)^2}\,dx + \frac{xy^2}{(x^2+y^2)^2}\,dy$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

small circle

$$x^2+y^2=\varepsilon^2$$

$x = \varepsilon \cos t$
$y = \varepsilon \sin t.$

$$\frac{\cancel{\varepsilon^4}}{\cancel{\varepsilon^4}} \int -\sin^3(t)(-\sin t\,dt) + \cos(t)\sin^2(t)\cos(t)\,dt$$

$$= \int_0^{2\pi} \left(\sin^4(t) + \sin^2(t)\cos^2(t)\right) dt$$

$$= \int_0^{2\pi} \sin^2(t)\,dt = \pi$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$$

Simply connected region D in plane

$$\oint_{\partial D} P\,dx + Q\,dy = \iint_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\int_{\theta_0}^{\theta_0+2n\pi} \sin^2(t)\,dt = \int_{\theta_0}^{\theta_0+2n\pi} \cos^2(t)\,dt$$

$= \frac{1}{2} \cdot$ length of interval

$$\iint (3x - 2y + z)$$

over part of the plane

$$2x + 3y + z = 6$$

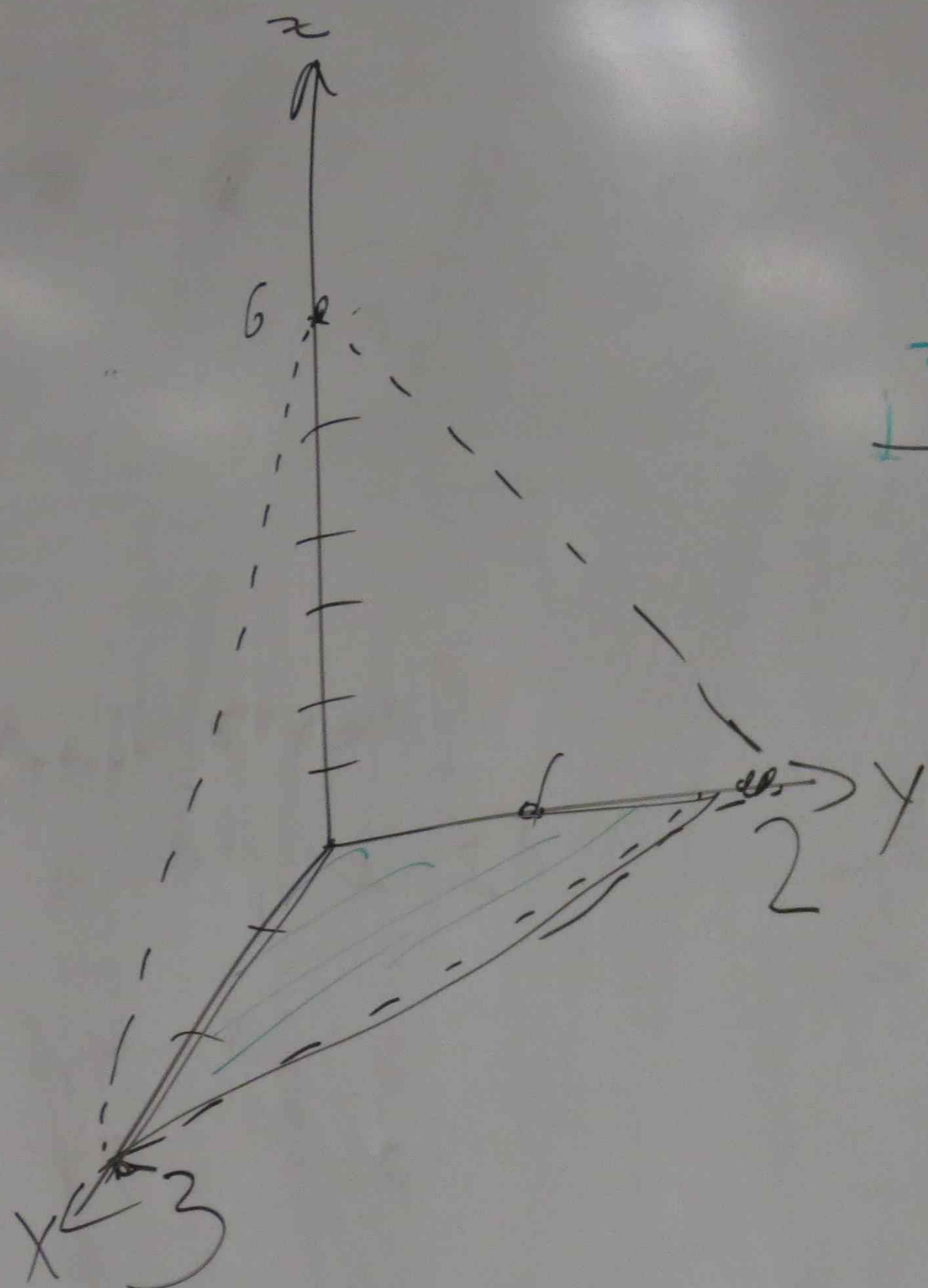
in 1st octant $(x \geq 0, y \geq 0, z \geq 0)$

$$z = 6 - 2x - 3y$$

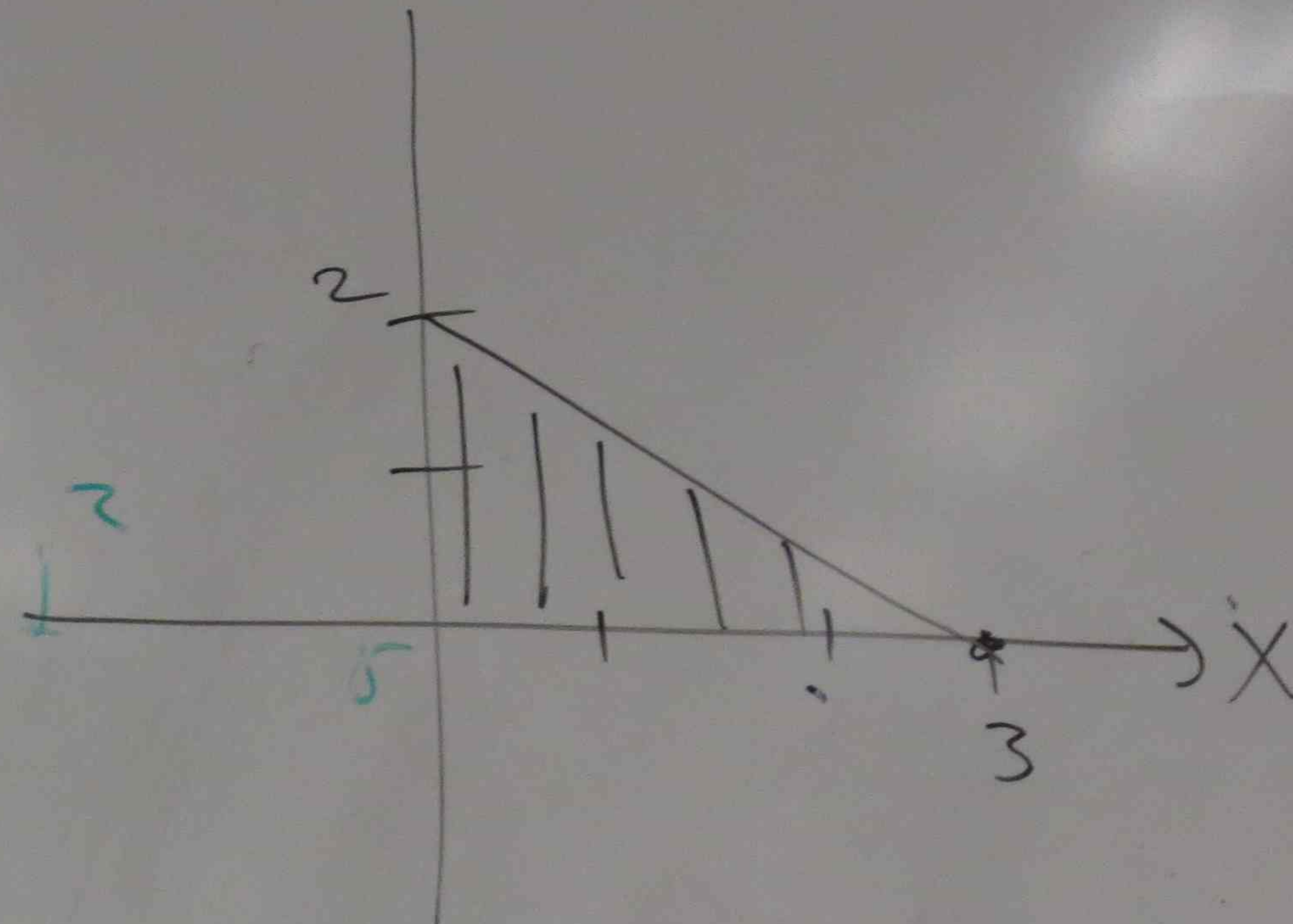
$$= f(x, y)$$

$$dS = \sqrt{1 + |\nabla f|^2}\, dx\,dy$$

$$\sqrt{14}\, dx\,dy$$



z
6
y
2
x
3



2
x
3

① Integrat on X-axis

from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} (-y)\, dx$$

②

$$\int_{\text{upper arc}} (-y)\, dx = -\int_{2\pi}^{0} y(\theta)\, dx(\theta) \cdots$$

$$= +a^2 \int_0^{2\pi} (1-\cos\theta)(1-\cos\theta)\, d\theta$$

defines $\mathcal{C}$.

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

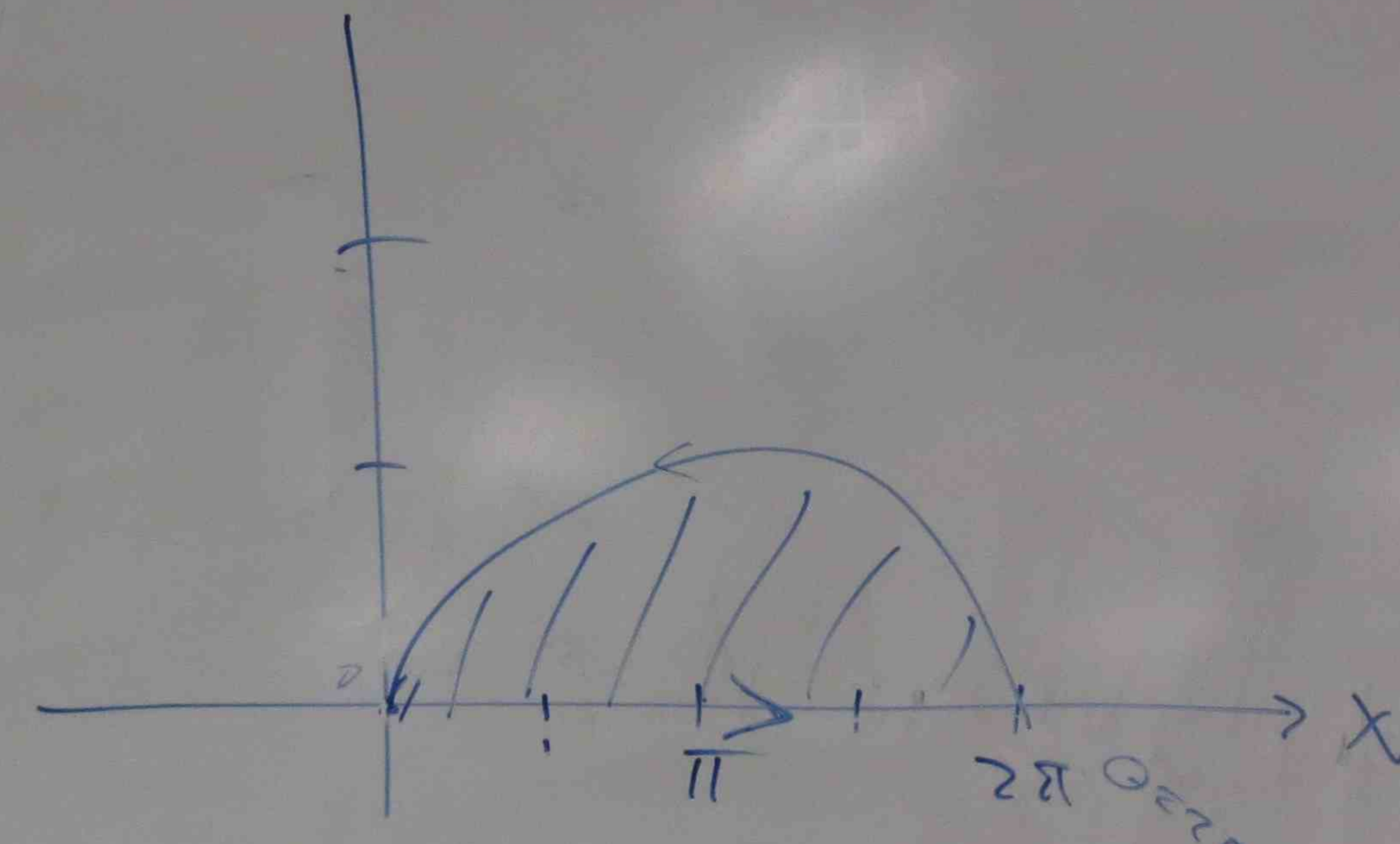
$$x = 0$$

$$\boxed{A = -\oint y\,dx}$$

also

$$= \oint x\,dy$$

also $= \frac{1}{2}\oint(-y\,dx + x\,dy)$



Integrals ccw.

$$\oint P\,dx + Q\,dy = \iint \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{1} dx\,dy$$

If $P = -y,\ Q = 0 \Rightarrow 1$