MATH 1502 EXAM #1 NAME: ______________________________

February 10, 2005

Answer the following questions clearly and completely. Unless otherwise specified, you must provide work justifying your solution. Use the back of the page for scratchwork if needed, but clearly indicate if anything on the back of the page should be graded, otherwise it will be ignored.

There are 5 questions plus one extra credit problem, on 7 pages. The exam is worth 40 points total, plus up to 3 points extra credit.

(8 points) 1. Determine whether the following infinite series converges.

$$\sum_{k=1}^{\infty} \frac{2k+1}{\sqrt{k^4+k}}$$

(8 points) 2. If the following improper integral converges, find its exact value.

$$\int_{-\infty}^{0} xe^{x}\,dx$$

(8 points) 3. Determine the radius and interval of convergence for the following power series.

$$\sum_{k=1}^{\infty} \frac{(-1)^k (k-1)}{k 2^k} x^k$$

4. Let $f(x) = \ln(1-x)$.

(3 points) a. Find a formula for $f^{(k)}(x)$ for $k = 0, 1, 2, 3, \ldots$

(1 point) b. Find $f^{(k)}(0)$ for $k = 0, 1, 2, \ldots$

(Problem 4 Continued)

(2 points) c. Find the nth Taylor polynomial $P_n(x)$ for f expanded about the point $x = 0$.

(5 points) d. Find (with proof) a value for x and n such that $P_n(x)$ approximates $\ln(0.8)$ to within an error of at most 0.01.

(5 points) 5. If the following infinite series converges, find its exact value:

$$\sum_{k=1}^{\infty} \frac{3^{k-1}}{5^{2k}}$$

(3 points EXTRA CREDIT) Suppose that $f(x) = \sum_{k=0}^{\infty} a_k x^k$ defines a power series that has radius of convergence $r > 0$. Show that if this power series is absolutely convergent at one endpoint of its interval of convergence, then it is absolutely convergence at the other endpoint.