MATH 1502 EXAM #2 NAME: ______________________________

March 10, 2005

Answer the following questions clearly and completely. Unless otherwise specified, you must provide work justifying your solution. Use the back of the page for scratchwork if needed, but clearly indicate if anything on the back of the page should be graded, otherwise it will be ignored.

There are 5 questions plus one extra credit problem, on 7 pages. The exam is worth 40 points total, plus up to 3 points extra credit.

1. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 2 & -2 \\ 0 & 1 \end{bmatrix}$. Compute the following quantities if possible, otherwise mark them as "undefined." No explanation required, just write down the answer.

(2 points) a. $BA =$

(3 points) b. $B^{\mathrm{T}}A =$

2. This question is about the following system of linear equations:

$$\begin{aligned} x_1 + 3x_2 - 4x_5 &= -1 \\ -2x_1 - 6x_2 - x_3 + 8x_5 &= 5 \\ x_1 + 3x_2 + x_3 + x_4 &= 2 \end{aligned}$$

(3 points) a. Find a matrix A, a vector x, and a vector b so that the system above is equivalent to the vector equation $Ax = b$. (No explanation required.)

(6 points) b. Determine if the system is consistent. If so, write the general solution in vector parametric form.

(Problem 2 Continued)

(1 point) c. Give the reduced row echelon form R of A (NOT augmented). No work required; you should have done it already in part b.

(2 points) c. Give elementary matrices $E_1, \ldots, E_k$ (how many depends on how you do the problem) such that $R = E_k \cdots E_1 A$.

(2 points) d. Do the columns of A span $\mathbf{R}^3$? How do you know?

(Problem 2 Continued)

(2 points) e. Are the columns of A linearly independent? How do you know?

(2 points) f. What is the rank and nullity of A? No explanation required.

(2 points) g. Is b a linear combination of the columns of A? If so, give one specific linear combination of the columns that equals b.

3. Suppose that A is some 4×8 matrix.

(2 points) a. Which one of the following statements is true, and why? (Short explanation.)

A. A has 8 columns, and the columns form a set of dependent vectors in $\mathbf{R}^4$.
B. A has 4 columns, and the columns form a set of dependent vectors in $\mathbf{R}^8$.
C. A has 8 columns, and the columns form a set of independent vectors in $\mathbf{R}^4$.
D. A has 4 columns, and the columns form a set of independent vectors in $\mathbf{R}^8$.

(2 points) b. Suppose that A is some 4×8 matrix and that $b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

Which one of the following statements is true, and why? (Short explanation.)

A. If $\text{rank}(A) = 4$ then $Ax = b$ has a unique solution.
B. If $\text{rank}(A) = 4$ then $Ax = b$ has infinitely many solutions.
C. If $\text{rank}(A) = 4$, there's not enough information to determine whether $Ax = b$ has a solution.

4. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 2 & -1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(6 points) a. Find the inverse of A, if it exists.

(1 point) b. Use your answer from part a to solve the equation $Ax = b$.

(4 points) 5. For each part, circle T for True or F for False (no explanation required). Note: An answer of "sometimes true and sometimes false" or "cannot be determined" would count as False.

T F a. If A and B are invertible $n \times n$ matrices, then AB is an invertible $n \times n$ matrix, and $(AB)^{-1} = A^{-1}B^{-1}$.

T F b. If the last row of the reduced row echelon form of an augmented matrix of a system of linear equations has only zero entries, then the system has infinitely many solutions.

T F c. If A is an $m \times n$ matrix and $b \in \mathbf{R}^m$, then the equation $Ax = b$ is consistent if and only if b is a linear combination of the columns of A.

T F d. If $v_1, \ldots, v_k$ is a dependent set of vectors in $\mathbf{R}^5$, then $k > 5$.

(3 points EXTRA CREDIT) Suppose that A is an $m \times n$ matrix and B is an $n \times m$ matrix, with $m > n$. Prove that AB can never equal the $m \times m$ identity matrix I_m.