

PLEASE READ THESE DIRECTIONS: Answer PROBLEM 1 (15 points) and choose TWO other problems to answer (10 points each). You may also answer (for up to 3 points extra credit) ONE additional problem. In this case, please specify which problem is the extra credit problem.

All statements require proof or justification. There are 35 points total, plus up to 3 points of extra credit.

1. Let $A(\mathbb{R})$ be the set of all bijections of the real line \mathbb{R} onto itself. We know that this is a group under the operation of composition of functions (you do not need to prove that fact).

a. Identify three different functions that belong to $A(\mathbb{R})$, i.e., give a formula for three functions that belong to $A(\mathbb{R})$.

Note: You don't need to prove that your functions are bijections, you just need to give formulas for three functions that are bijections.

b. Identify the identity element of $A(\mathbb{R})$, i.e., give the formula for the function that is the identity of $A(\mathbb{R})$. Prove that your function is indeed the identity element of $A(\mathbb{R})$.

c. Let $f(x) = x + 1$. Determine (with proof) whether f has finite order or not.

2. Let G_1 and G_2 be groups. Suppose that $f: G_1 \rightarrow G_2$ is a homomorphism and H is a subgroup of G_2 . Prove that $f^{-1}(H) = \{a \in G_1 : f(a) \in H\}$ is a subgroup of G_1 .

3. Let G, H be groups and let $f: G \rightarrow H$ be an isomorphism. Prove that if an element $a \in G$ has finite order $o(a)$, then the order of $f(a)$ in H is $o(f(a)) = o(a)$.

4. Suppose that G is a cyclic group and N is a subgroup of G . Prove that G/N is cyclic. Hint: Why do you know that N is normal?