Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 4.2 #8.

2. Problem 4.4 #6.

Note: You may assume without proof that R is a subring of $\mathbb{R}$, but be sure to show that M is an ideal in R.

Hints for showing M is maximal: Suppose that I is an ideal such that $M \subseteq I \subseteq R$. You have to show that either $I = M$ or $I = R$. If it was the case that $I = M$ then you're done, so suppose that $I \neq M$. In this case, there exists some element $k + \ell\sqrt{2}$ that belongs to I but not to M. Consider $(k + \ell\sqrt{2})\,(k - \ell\sqrt{2})$; this belongs to what ideal? And then write $k = 5m + r$ and $\ell = 5n + s$ where r and s are the remainders after dividing by 5.

3. Problem 4.4 #7. With M and R as in Problem 4.4 #6, show that R/M is a field having 25 elements.

4. Problem 4.5 #14, part a only.