1. Prove the following statements. Note: Part a is not related to parts b or c of this problem.
   a. Show that \( \inf \left\{ \frac{n + 2}{n + 1} : n \in \mathbb{N} \right\} = 1. \)
   Hint: First show that 1 is a lower bound, and then show that it is the greatest lower bound.
   b. Let \( A \) and \( B \) be subsets of \([0, \infty)\) that are bounded above. Define the set \( AB \) to be
      \[ AB = \{ ab : a \in A, b \in B \}. \]
      Prove that \( AB \) is bounded above, and that
      \[ \sup(AB) \leq \sup(A) \sup(B). \]
   c. In part b we required \( A \) and \( B \) to only contain nonnegative numbers. For this part, let \( A \)
      and \( B \) be any bounded subsets of \( \mathbb{R} \). Either prove that
      \[ \sup(AB) = \sup(A) \sup(B), \]
      or find an example of sets \( A \) and \( B \) for which equality does not hold.

2. a. Let \( A \) and \( B \) be two denumerable sets, such that \( A \cap B = \emptyset \). Show that \( A \cup B \)
      is denumerable by exhibiting a \( 1 \rightarrow 1 \) correspondence between \( A \cup B \) and \( \mathbb{N} \).
   b. Let \( X \) be an uncountable set, and let \( Y \) be a denumerable subset of \( X \). Show that \( X \setminus Y \)
      is uncountable. Hint: use part a.
   c. Prove that \( \{ x \in \mathbb{R} : x \text{ is irrational} \} \) is uncountable.

3. a. Let \( X, Y, Z \) be sets, and suppose that \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) are functions such that
      \( g \circ f : X \rightarrow Z \) is a bijection. Show that \( f \) is injective and \( g \) is surjective.
   b. Give an example of sets \( X, Y, Z \) and functions \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) such that
      \( g \circ f : X \rightarrow Z \) is a bijection but \( f \) is not a surjection and \( g \) is not an injection.

4. Let \( A \subseteq \mathbb{R}^p \) be given, and define the \textit{interior} of \( A \) to be the set
   \[ A^o = \{ x \in \mathbb{R}^p : x \text{ is an interior point of } A \}. \]
   a. Prove that if \( U \) is an open set and \( U \subseteq A \), then \( U \subseteq A^o \).
   b. Prove that if \( A, B \) are two subsets of \( \mathbb{R}^p \), then \( (A \cap B)^o = A^o \cap B^o \).