Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 7 G. If \(I_n = [a_n, b_n], \ n \in \mathbb{N}\), is a nested sequence of closed cells, show that for every \(m\) and \(n\) we have
   \[a_1 \leq a_2 \leq \cdots \leq a_n \leq \cdots \leq b_m \leq \cdots \leq b_2 \leq b_1.\]
   If we put \(\xi = \sup\{a_n : n \in \mathbb{N}\}\) and \(\eta = \inf\{b_m : m \in \mathbb{N}\}\), show that \([\xi, \eta] = \bigcap_{n \in \mathbb{N}} I_n\).

2. Let \(F\) be the Cantor set constructed in class. Prove that there is no interval \((a, b)\) that is contained in \(F\). Specifically, show that if \(0 < a < b < 1\), then \((a, b)\) is not a subset of \(F\).

3. Problem 8 L (refer to 8 F and 8 G for definitions). Show that there exist positive constants \(a, b\) such that
   \[a \|x\|_1 \leq \|x\|_\infty \leq b \|x\|_1\]
   for all \(x \in \mathbb{R}^p\).
   Find the largest constant \(a\) and the smallest constant \(b\) with this property.

4. Problem 8 Q. A subset \(K\) of \(\mathbb{R}^p\) is said to be convex if, whenever \(x, y\) belong to \(K\) and \(t\) is a real number such that \(0 \leq t \leq 1\), then the point
   \[(1 - t)x + ty = x + t(y - x)\]
   also belongs to \(K\). Interpret this condition geometrically and show that the subsets
   \[K_1 = \{x \in \mathbb{R}^2 : \|x\| \leq 1\},\]
   \[K_2 = \{\langle \xi, \eta \rangle \in \mathbb{R}^2 : 0 < \xi < \eta\},\]
   \[K_3 = \{\langle \xi, \eta \rangle \in \mathbb{R}^2 : 0 \leq \eta \leq \xi \leq 1\},\]
   are convex but that the subset
   \[K_4 = \{x \in \mathbb{R}^2 : \|x\| = 1\}\]
   is not convex.