1. Problem 9 L. If $A$ is any subset of $\mathbb{R}^p$, let $A^-$ denote the intersection of all closed sets containing $A$; the set $A^-$ is called the **closure** of $A$. Note that $A^-$ is a closed set; prove that it is the smallest closed set containing $A$. Prove that

$$A \subseteq A^-, \quad (A^-)^- = A^-, \quad (A \cup B)^- = A^- \cup B^-,$$

$\emptyset^- = \emptyset$.

Give an example to show that $(A \cap B)^- = A^- \cap B^-$ may not hold.

**NOTE:** Part of this problem asks you to prove that $A^-$ is the smallest closed set containing $A$. To do this, you must show that $A^-$ is closed, and that if $B$ is any closed set such that $A \subseteq B$, then $A^- \subseteq B$.

2. Problem 10 G. Show that every point in the Cantor set $F$ is a cluster point of both $F$ and $C(F)$.

3. Problem 11 D. Prove that if $K$ is a compact subset of $\mathbb{R}$, then $K$ is compact when regarded as a subset of $\mathbb{R}^2$.

**NOTES:** The statement “when $K$ is regarded as a subset of $\mathbb{R}^2$” means that you are to prove that the set

$$K' = \{ (x,0) : x \in K \}$$

is a compact subset of $\mathbb{R}^2$. Do NOT use the Heine–Borel Theorem in your proof; instead, prove that $K'$ is compact by directly using the definition of compact set.

4. Problem 12 B. If $C \subseteq \mathbb{R}^p$ is connected and $x$ is a cluster point of $C$, then $C \cup \{x\}$ is connected.