

## 19. Some Extensions

### Definition

Let  $(x_n)$  &  $(y_n)$  be sequences of real nos,  $y_n \neq 0 \forall n$  large enough

a.  $(x_n)$  is equivalent to  $(y_n)$ , written  $(x_n) \sim (y_n)$ , if

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = 1.$$

b.  $(x_n)$  is a lower order of magnitude than  $(y_n)$  if

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = 0$$

Write  $x_n = o(y_n)$

c.  $(x_n)$  is dominated by  $(y_n)$  if  $(\frac{x_n}{y_n})$  is bounded:

~~Write  $x_n = O(y_n)$~~

$$\exists A, B \text{ st. } A \leq \frac{x_n}{y_n} \leq B \forall n. \quad \text{Write } x_n = O(y_n).$$

## 11. Switching Order

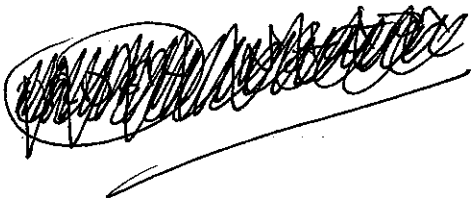
Can't switch limits, or sums & limits, or integrals & limits, in general.

Need not be true that

$$\lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} a_{m,n} = \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} a_{m,n}$$

$$\sum_{n=1}^{\infty} \left( \lim_{n \rightarrow \infty} a_{m,n} \right) = \lim_{n \rightarrow \infty} \left( \sum_{m=1}^{\infty} a_{m,n} \right)$$

etc.



Fatou for series  $\sum_n a_{m,n} \geq 0 \forall m, n$  then

$$\sum_m \limsup_{n \rightarrow \infty} (a_{m,n}) \leq \limsup_{n \rightarrow \infty} \left( \sum_m a_{m,n} \right)$$

Ex:  $1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \dots$

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \dots$

Row  $m$   $\frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32} \quad \dots \rightarrow \limsup_{n \rightarrow \infty} a_{m,n} = 0 \rightarrow \sum_m \limsup a_{m,n} = 0$

↓

Column  $n$

$$\sum_{m \geq 1} a_{m,n} = 2^{-n} \quad \limsup \left( \sum_m a_{m,n} \right) = 0$$

But

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \dots$

$0 \quad 1 \quad \frac{1}{2} \quad \frac{1}{4} \rightarrow \limsup_{n \rightarrow \infty} a_{m,n} = 0$

$\sum_m \limsup a_{m,n} = 0$

$0 \quad 0 \quad 1 \quad \frac{1}{2}$

$\vdots$

↓

$$\limsup_{n \rightarrow \infty} \left( \sum_m a_{m,n} \right) = 1$$

*RAMS with big numbers, etc.*