MATH 6337 HOMEWORK #7 Due date: November 13, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 3.1 #2. (a) Show that if ν is a signed measure, then

$$E \text{ is } \nu\text{-null} \quad \Longleftrightarrow \quad |\nu|(E) = 0.$$

(b) Show that if μ, ν are signed measures, then

$$\nu \perp \mu \quad \Longleftrightarrow \quad |\nu| \perp \mu \quad \Longleftrightarrow \quad \nu^+, \nu^- \perp \mu.$$

2. Problem 3.1 #3. Let ν be a signed measure on $(X, \mathcal{M})$.

(a) Show that $L^1(\nu) = L^1(|\nu|)$.

(b) Show that if $f \in L^1(\nu)$, then $|\int f\,d\nu| \le \int |f|\,d|\nu|$.

Remark: We showed in class how the complex-valued case follows from the real-valued case, so you just have to do the real-valued case.

(c) Show that if $E \in \mathcal{M}$, then

$$|\nu|(E) \;=\; \sup\biggl\{\biggl|\int_E f\,d\nu\biggr| : |f| \le 1\biggr\}.$$

3. Problem 3.1 #7. Let ν be a signed measure on $(X, \mathcal{M})$, and choose $E \in \mathcal{M}$. Show that

$$\nu^+(E) \;=\; \sup\bigl\{\nu(A) : A \in \mathcal{M},\ A \subseteq E\bigr\},$$

$$\nu^-(E) \;=\; -\inf\bigl\{\nu(A) : A \in \mathcal{M},\ A \subseteq E\bigr\},$$

$$|\nu|(E) \;=\; \sup\biggl\{\sum_{k=1}^{n} |\nu(E_k)| : n \in \mathbb{N},\ E_k \in \mathcal{M},\ E = \bigcup_{k=1}^{n} E_k \text{ disjointly}\biggr\}.$$