
Preface

This volume celebrates the work of John Joseph Benedetto. Mathematician, mentor, father, friend—John has had a profound influence not only on the direction of harmonic analysis and its applications, but also on the entire community of people involved in these endeavors. This volume collects articles from coauthors, students, and colleagues of John, representative of some of the major areas that John has contributed to, including harmonic analysis, frame theory, time-frequency analysis, wavelet theory, and sampling theory and shift-invariant spaces. In recognition of John’s own high standards of mathematical exposition, and in order to create a volume of lasting utility, many of the articles in this volume include introductions to or surveys of their representative research directions.

As is clearly illustrated in Hans Feichtinger’s foreword to this volume, John’s work has covered an enormous breadth of areas, and space limitations have required us to concentrate on the few representative themes mentioned above. Of course, this separation into parts is inherently artificial, as all these directions interrelate in many ways—they are each simply different facets of the vast field that we call harmonic analysis.

We begin in Part I with a tribute to John’s early (and continuing) work in classical harmonic analysis. Among the many topics he has worked on in this area are weighted-norm inequalities for the Fourier transform, generalized harmonic analysis, Fourier restriction theorems, uncertainty principles, spectral synthesis, and the interaction between harmonic analysis and number theory. The Gibbs phenomenon (first observed by Wilbraham in 1848) is a fundamental fact in the theory of Fourier series in one dimension. The article by George Benke (who was the first of John’s many Ph.D. students) explores Gibbs phenomena in higher dimensions, where the multitude of possible types of discontinuities makes the analysis far more intricate and interesting. Hans Heinig is a longtime coauthor of John, with their first joint paper appearing in 1983. His article uses Fourier inequalities between weighted Lebesgue spaces to derive weighted Sobolev gradient inequalities for a wide range of indices. The final article in this part is by John’s 17th Ph.D. student, Georg Zimmer-

mann. It explores semidiscrete multipliers, describing their basic form and deriving representations of such multipliers for a number of important spaces.

In Part II we turn to abstract frame theory. It is not an understatement to say that John's 2003 paper *Finite normalized tight frames* (Adv. Comput. Math., Vol. 18, pp. 357–385), written jointly with his 24th Ph.D. student, Matthew Fickus, revolutionized the theory of frames in finite dimensions. The article by Peter Casazza, Matthew Fickus, Jelena Kovačević, Manuel Leon (John's 20th Ph.D. student), and Janet Tremain surveys and further develops this theory, proving a striking fundamental inequality that is both necessary and sufficient for the existence of finite frames with prescribed norms.

Frame theory, time-frequency analysis, and wavelet theory are all closely intertwined, and John has made fundamental contributions to all of these areas. In Part III we focus on time-frequency analysis (also known as Gabor analysis). John's work on the *Balian–Low Theorems* includes symplectic BLTs in higher dimensions and sharp versions of the BLT. The current state-of-the-art of the Balian–Low Theorems is surveyed in an article by two of his collaborators on this subject, Wojciech Czała (a former postdoc) and Alexander Powell (John's 29th Ph.D. student). The Zak transform is a fundamental tool in time-frequency analysis and in the analysis and application of Gabor frames. The article by John's 6th Ph.D. student, Jean-Pierre Gabardo, studies and surveys the Zak transform of distributions, and derives necessary and sufficient conditions for the finite linear span of a Gabor system to be dense in the Schwartz space of tempered distributions.

Because of the Balian–Low Theorems, it is *redundant* Gabor frames that must be used in practice. Necessarily, such redundant frames have nonunique dual frames. The article by Eric Hayashi, Shidong Li (John's 15th Ph.D. student), and Tracy Sorrells surveys the existing characterizations of Gabor dual frames, showing the equivalence of each characterization.

Much of the modern theory of quantitative time-frequency analysis, and in particular the introduction, development, and application of the modulation spaces, is due to John's longtime friends Hans Feichtinger and Karlheinz Gröchenig. In his article, Gröchenig surveys the relationship and application of the modulation spaces to the theory of pseudodifferential operators. Today, pseudodifferential operators are finding a wide variety of new applications in wireless communication, medical imaging, and geophysics, and these applications require new tools and techniques, which time-frequency analysis is well suited to deliver.

A basic, simple-to-state, and yet still open question regarding Gabor systems is the following: *are Gabor systems finitely linearly independent?* This is unknown in general even for the case of a collection of four time-frequency shifts of a single L^2 function. How hard can it be to prove that four functions in $L^2(\mathbf{R})$ must be independent? My article surveys this and related problems in Gabor theory.

Part IV is devoted to wavelet theory. In 1997, in an article in the journal *Journal of Fourier Analysis and Applications* (which had been founded by

John only three years before), Xingde Dai, David Larson, and Darrin Speegle surprised the wavelet community by proving the existence of “wavelet sets.” The characteristic function of such a set is a single function that generates an orthonormal wavelet basis for $L^2(\mathbf{R}^n)$, whereas wavelet orthonormal basis in higher dimensions generated from multiresolution analyses necessarily require multiple generators when the dilation matrix A satisfies $|\det(A)| > 2$. Together with Manuel Leon (his 20th Ph.D. student), and Songkiat Sumetkijakan (his 26th Ph.D. student), John has written several influential papers on wavelet sets. One of the first wavelet sets constructed by Dai, Larson, and Speegle was the “wedding cake wavelet set.” Pictured on the front cover of this volume is one of the wavelet sets constructed by John and Songkiat. This one is John’s favorite, and is known as the “wedding night wavelet set.”

The first article in Part IV, by David Larson, Eckart Schulz, Darrin Speegle, and Keith Taylor, represents abstract wavelet theory, with an article on explicit cross sections of singly generated group actions. These cross sections are used to characterize those matrices for which there exist minimally supported frequency (MSF) wavelets with infinitely many wavelet generators.

I cannot resist expressing my own personal satisfaction at being able to include the second article in this part, by Kangui Guo, Demetrio Labate, Wang-Q Lim, Guido Weiss, and Edward Wilson. This article surveys a recent development in wavelet constructions in higher dimensions that employs several dilation matrices. This allows the construction of wavelet bases which are well suited for representing the multitude of geometric features that can occur in higher dimensions. Guido has been a friend and mentor to generations of mathematicians, including John and his students in particular, not to mention that he been one of the cornerstone figures of harmonic analysis over the last 50 years. Demetrio is my own very first Ph.D. student, and so is John’s “mathematical grandchild.” This article represents the continuing impact of stellar mathematicians such as Guido and John on the newest generations of mathematicians.

As noted in Hans Feichtinger’s foreword, John’s contributions have ranged far across both pure mathematics and applications. Indeed, much of his work shows how naturally harmonic analysis belongs with applications. His Wavelet Auditory Model (WAM), developed jointly with his 14th Ph.D. student, Anthony Teolis, incorporates his work on irregular sampling to build a model of the human cochlear response. Their 1993 WAM article was the first paper in the first issue of the then-new and now-major journal *Applied and Computational Harmonic Analysis*. John’s work on regular and irregular sampling, and the closely related area of shift-invariant spaces, is honored in the final portion, Part V, of this volume.

The first article in this section is by Jeffrey Hogan and Joseph Lakey (John’s 12th Ph.D. student). It discusses the important special cases of sampling sets that are not lattices but that still possess a periodic structure, e.g., $\mathbb{Z} \cup (\mathbb{Z} + \frac{1}{\pi})$. These types of sampling sets arise in many real-world settings. The article by Bjarte Rom and John’s 7th Ph.D. student, David Walnut, addresses

another kind of irregular but still somewhat structured sampling geometry. In their setting the sample set is not necessarily well-separated, and thus is outside the bounds of most of the sampling literature. Yet the specific structure addressed, which is a union of lattices (such as $\mathbb{Z} \cup \sqrt{2}\mathbb{Z}$), allows many interesting and useful conclusions to be drawn.

In their article, John's longtime friends Akram Aldroubi, Carlos Cabrelli, and Ursula Molter use the setting of shift-invariant spaces to survey recent results relating frame theory and shift-invariant spaces to learning theory. Larry Baggett, another longtime friend, analyzes the fine structure of shift-invariant spaces. His article, which includes an elegant introduction to frame theory, shows how a certain multiplicity function can be used to quantify the notion of redundancy of frequencies.

The final article, by Peter Casazza, Ole Christensen, Alex Lindner, and Shidong Li, addresses the delicate question of characterizing frame properties of sets of complex exponentials based solely on the density of the set of frequencies of those exponentials.

Although our survey of the articles appearing in this volume is now complete, as a tribute to John's work this volume is certainly far from complete. In the "pure" directions we could well have included articles on generalized harmonic analysis, uncertainty principles, spectral synthesis, and number theory, to name only a few. In the "applied" direction we have omitted mention of John's fundamental work on Σ - Δ quantization, spectral estimation, periodicity detection, EEGs, MRIs—and again this list is incomplete. Sadly, space has limited the number of articles and the number of contributors to a small subset of those who would have eagerly contributed their own tribute to John.

This volume is also incomplete in that it focuses on research alone, while John's contributions go far beyond this. In particular, I am honored to be one of the many Ph.D. students whom John has mentored. Listed on the next two pages is John's mathematical genealogy as it stands at this moment, including his Ph.D. students and grandstudents, his Ph.D. advisor Chandler Davis, and Chandler's advisor Garrett Birkhoff. In the backward direction the graph ends there, for though Garrett Birkhoff was a professor at Harvard, he had no formal Ph.D. degree. In the forward direction, the graph is still far from complete even in listing John's students, as he is still actively advising an entire group of current students. John's dedication to all facets of mathematics, including his research, his Ph.D. students, and all the many undergraduate and graduate students who have taken courses from him, were recognized in 1999 when he was named a Distinguished Scholar/Teacher of the University of Maryland, College Park.

I am sure that all of John's many friends will join with me to wish him health, happiness, and many more years of successful and significant mathematics.

Atlanta, Georgia
March 2006

Christopher Heil

The Benedetto Mathematical Family Tree

- Garrett Birkhoff (no Ph.D.)
 - Chandler Davis (Harvard U.), 1950
 - John J. Benedetto (U. Toronto), 1964
 - George Benke (U. Maryland, College Park), 1971
 - Wan-chen Hsieh (U. Maryland, College Park), 1977
 - Fulvio Ricci (U. Maryland, College Park), 1977
 - Giovanna Carcano (U. Milano), 1985
 - Paolo Ciatti (U. Torino), 1996
 - Bianca di Blasio (U. Torino), 1996
 - Mario Dellanegra (U. Torino), 1998
 - Priscilla Gorelli (U. Torino), 1999
 - Silvia Secco (U. Torino), 1999
 - Daniele Debertol (U. Genova), 2003
 - Ward Evans (U. Maryland, College Park), 1980
 - William Joyner (U. Maryland, College Park), 1983
 - Jean-Pierre Gabardo (U. Maryland, College Park), 1987
 - Xiaojiang Yu (McMaster U.), 2005
 - David Walnut (U. Maryland, College Park), 1989
 - Paul Salamonowicz (George Mason U.), 1999
 - Glenn Easley (George Mason U.), 2000
 - Christopher Heil (U. Maryland, College Park), 1990
 - Demetrio Labate (Georgia Tech), 2000
 - Denise Jacobs (Georgia Tech), 2001
 - Kasso Okoudjou (Georgia Tech), 2003
 - Rodney Kerby (U. Maryland, College Park), 1990
 - George Yang (U. Maryland, College Park), 1990
 - William Heller (U. Maryland, College Park), 1991
 - Joseph Lakey (U. Maryland, College Park), 1991
 - Ying Wang (New Mexico State U.), 2001
 - Sofian Obeidat (New Mexico State U.), 2002
 - Christopher Weaver (New Mexico State U.), 2005

- Erica Bernstein (U. Maryland, College Park), 1992
- Anthony Teolis (U. Maryland, College Park), 1993
- Shidong Li (U. Maryland, Baltimore County), 1993
- Sandra Saliani (U. Maryland, College Park), 1993
- Georg Zimmermann (U. Maryland, College Park), 1994
- Melissa (Harrison) Rice (U. Maryland, College Park), 1998
- Hui-Chuan Wu (U. Maryland, College Park), 1998
- Manuel Leon (U. Maryland, College Park), 1999
- Götz Pfander (U. Maryland, College Park), 1999
- Oliver Treiber (U. Maryland, College Park), 1999
- Sherry Scott (U. Maryland, College Park), 2000
- Matthew Fickus (U. Maryland, College Park), 2001
- Ioannis Konstantinidis (U. Maryland, College Park), 2001
- Songkiat Sumetkijakan (U. Maryland, College Park), 2002
- Anwar Saleh (U. Maryland, College Park), 2002
- Jeffrey Sieracki (U. Maryland, College Park), 2002
- Alexander Powell (U. Maryland, College Park), 2003
- Shijun Zheng (U. Maryland, College Park), 2003
- Joseph Kolesar (U. Maryland, College Park), 2004
- Andrew Kebo (U. Maryland, College Park), 2005
- Juan Romero (U. Maryland, College Park), 2005
- Abdelkrim Bourouhiya (U. Maryland, College Park), 2006²
- Aram Tangboondouangjit (U. Maryland, College Park), 2006³

²To be awarded in May 2006.

³To be awarded in May 2006.