Elementary probability

Consider a procedure (often called an experiment) with a set of possible outcomes $S$. The set $S$ is called the sample space of the experiment, and a subset $A \subseteq S$ is called an event.

Assume that all outcomes in a finite sample space $S$ are equally likely. The probability of an event $A \subseteq S$ is denoted $P(A)$ and defined to be

$$P(A) = \frac{|A|}{|S|}.$$

Let $S$ be a (finite) sample space with events $A, B \subseteq S$ and $A_i \subseteq S$ for $1 \leq i \leq n$. Assume that all outcomes in $S$ are equally likely (a.k.a. “the probabilities are uniform”).

**Theorem 1**

1. $P(\emptyset) = 0$, $P(S) = 1$, and $0 \leq P(A) \leq 1$ in general.
2. $P(A^c) = 1 - P(A)$ since $A^c = S \setminus A$.
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

**Theorem 2**

$$P(\bigcup_{i=1}^{n} A_i) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \sum_{1 \leq i < j < k \leq n} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n+1}P(\bigcap_{i=1}^{n} A_i).$$

Two events $A, B \subseteq S$ are mutually exclusive if $A \cap B = \emptyset$. Multiple events $A_i \subseteq S$ are pairwise mutually exclusive if $A_i \cap A_j = \emptyset$ for $1 \leq i < j \leq n$.

**Corollary 1** Let $A_i$ be pairwise mutually exclusive events for $1 \leq i \leq n$. Then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \leq i \leq n} P(A_i).$$

How to solve uniform, finite probability questions

**Step 1:** Identify the sample space $S$ and event $A$.

**Step 2:** Find $P(A)$ by the appropriate counting method.