Let $A$ be the $m \times n$ constraint matrix of an LP in standard form with feasible region $S = \{x \mid Ax = b, x \geq 0\}$ for $b \geq 0$;

$$\max \ (\min) \ \{cx \mid x \in S\}.$$ 

Consider an $m \times m$ submatrix $B$ of $A$ consisting of $m$ linearly independent columns forming a basis. Let $x_B$ denote the variables associated with the columns of $B$. The solution $x_B = B^{-1}b$ to the system $Bx_B = b$ is called a basic solution of the system $Ax = b$.

**Yet more LP problem terminology**

**Basic feasible solution (BFS):** A basic solution $x_B$ with $x_B \geq 0$.

**Basic variable:** A variable in the BFS.

**Nonbasic variable:** A variable not in the BFS, necessarily has value 0.

**Degenerate BFS:** A BFS where some basic variable has value 0.

There are at most $\binom{n}{m}$ BFS for an LP in standard form.

**Overview of the Simplex Algorithm**

**Step 0:** Start with a BFS.

**Step 1:** Until an optimal solution is reached, move from one extreme point to another.

Bookkeeping is done using simplex tableau. Columns are (permanently) labeled by variables, except for the RHS value column. Objective function row is unlabeled. Constraint rows are labeled by current basic variables, one per equation.

Tableau columns labeled by current basic variables must be unit vectors. The intersection of the entering (currently nonbasic) variable column and exiting (currently basic) variable row is called the pivot cell of the current tableau. The sequence of elementary row operations which produces a new tableau for the entering variable is called pivoting.

**The Simplex Algorithm**

**Step 0:** Initialize the simplex tableau for a BFS (usually, the “all-slack” one).

**Step 1a:** Choose pivot column/entering variable by finding (nonbasic) $x_{j^*}$ with most negative coefficient (when maximizing) in objective function row.

**Step 1b:** Choose pivot row/exiting variable by finding (basic) $x_{i^*}$ such that $\frac{b_{i^*}}{a_{i^*j^*}} \leq \frac{b_i}{a_{ij}}$ for all $i$ with $a_{ij^*} > 0$.

**Step 1c:** Pivot on cell $(i^*, j^*)$ in row $x_{i^*}$ and column $x_{j^*}$.

**Step 1d:** Repeat until all coefficients in objective function row are nonnegative (for max).