Names:

Solutions are to be written on the board.

1. (a) Show that there is exactly one greatest element of a poset, if such an element exists.
   (b) Show that there is exactly one maximal element in a poset with a greatest element.
   (c) Show that the least upper bound of a set in a poset is unique if it exists.

2. Repeat Problem 1, replacing “greatest” by “least,” “maximal” by “minimal”, and “upper” by “lower.”

3. Let $(S, R)$ be a poset. Show that $(S, R^{-1})$ is also a poset, where $R^{-1}$ is the inverse of $R$.

4. Show that if the poset $(S, R)$ is a lattice, then the dual poset $(S, R^{-1})$ is also a lattice.

5. Suppose that $(S, \preceq_1)$ and $(T, \preceq_2)$ are posets. Show that $(S \times T, \preceq)$ is a poset where $(s, t) \preceq (u, v)$ if and only if $s \preceq_1 u$ and $t \preceq_2 v$ for $s, u \in S$ and $t, v \in T$.

6. Prove that every chain (also known as a totally ordered set) is a lattice.

7. Give an example of an infinite lattice (and justify your answer) with
   (a) neither a least nor a greatest element
   (b) a least but not a greatest element
   (c) a greatest but not a least element
   (d) both a least and a greatest element