Elementary probability

Consider a procedure (often called an experiment) with a set of possible outcomes \( S \). The set \( S \) is called the sample space of the experiment, and a subset \( A \subseteq S \) is called an event.

Assume that all outcomes in a finite sample space \( S \) are equally likely. The probability of an event \( A \subseteq S \) is denoted \( P(A) \) and defined to be

\[
P(A) = \frac{|A|}{|S|}.
\]

Let \( S \) be a (finite) sample space with events \( A, B \subseteq S \) and \( A_i \subseteq S \) for \( 1 \leq i \leq n \).

Assume that all outcomes in \( S \) are equally likely (a.k.a. “the probabilities are uniform”).

**Theorem 1**

1. \( P(\emptyset) = 0 \), \( P(S) = 1 \), and \( 0 \leq P(A) \leq 1 \) in general.
2. \( P(A^c) = 1 - P(A) \) since \( A^c = S \setminus A \).
3. \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \).

**Theorem 2**

\[
P\left( \bigcup_{i=1}^{n} A_i \right) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \sum_{1 \leq i < j < k \leq n} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n+1} P\left( \bigcap_{i=1}^{n} A_i \right).
\]

Two events \( A, B \subseteq S \) are mutually exclusive if \( A \cap B = \emptyset \). Multiple events \( A_i \subseteq S \) are pairwise mutually exclusive if \( A_i \cap A_j = \emptyset \) for \( 1 \leq i < j \leq n \).

**Corollary 1** Let \( A_i \) be pairwise mutually exclusive events for \( 1 \leq i \leq n \). Then

\[
P\left( \bigcup_{i=1}^{n} A_i \right) = \sum_{1 \leq i \leq n} P(A_i).
\]

How to solve uniform, finite probability questions

**Step 1:** Identify the sample space \( S \) and event \( A \).

**Step 2:** Find \( P(A) \) by the appropriate counting method.