Repetitions

The number of ways to put \( k \) identical marbles into \( n \) boxes is \( \binom{n+k-1}{k} = \binom{n-1+k}{k} \).

Recall...

- The number of \( k \) permutations of \( n \) symbols is \( n(n-1)(n-2) \ldots (n-k+1) = \frac{n!}{(n-k)!} \).
- The number of \( k \) combinations of \( n \) symbols is \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \).

The number of ways to put \( k \) marbles into \( n \) numbered boxes:

<table>
<thead>
<tr>
<th></th>
<th>All same colored marbles</th>
<th>All different colored marbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>At most one per box</td>
<td>( \binom{n}{k} = \frac{n!}{(n-k)!k!} ) when ( 0 \leq k \leq n )</td>
<td>( \frac{n!}{(n-k)!} ) when ( 0 \leq k \leq n )</td>
</tr>
<tr>
<td></td>
<td>0 when ( k &gt; n )</td>
<td>0 when ( k &gt; n )</td>
</tr>
<tr>
<td>Any number per box</td>
<td>( \binom{n+k-1}{k} )</td>
<td>( n^k )</td>
</tr>
</tbody>
</table>

**NOTE:** Table 7.3 on page 233 is NOT CORRECT.

Derangements

A derangement of \( n \) distinct ORDERED symbols is a permutation in which no symbol is in its correct/original position. The number of derangements of \( n \) is denoted \( D_n \).

The number of derangements of \( n \geq 1 \) distinct ordered symbols is

\[
D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots (-1)^n \frac{1}{n!}).
\]

Recall that the Taylor series expansion for \( e^x \) is

\[
e^x = 1 + \frac{x}{1!} + \frac{x}{2!} + \frac{x}{3!} + \ldots.
\]

Hence the number of derangements can be APPROXIMATED as

\[ D_n \approx n! e^{-1}. \]