Mathematical optimization terminology

Objective function: The function being optimized/maximized (resp. minimized).

Constraints: A set of equalities and inequalities that solutions must satisfy.

Feasible region: The solution set of vectors satisfying the constraints.

Feasible solution: A vector in the feasible region.

Optimal solution: A feasible solution attaining largest (resp. smallest) objective value.

Linear programming (LP): All functions are linear; all variables are continuous.

Let $x = (x_1, \ldots, x_n)$ be an $n \times 1$ vector of variables. For parameters $c_j, a_{ij},$ and $b_i$ with $1 \leq i \leq m, 1 \leq j \leq n,$ let $c$ be an $1 \times n$ vector, $A$ an $m \times n$ matrix, and $b$ an $m \times 1$ vector. Parameters are LP problem coefficients, and $b_i$ values are right-hand-sides (RHS).

An LP problem in canonical form has max obj. func., $\leq$ constraints, and nonneg. $x_j$:

$$\max z = cx \text{ such that } Ax \leq b \text{ and } x \geq 0.$$ 

An LP problem in standard form has nonneg. RHS values, $=$ constraints, and nonneg. $x_j$:

$$\max (\min) z = cx \text{ such that } Ax = b \text{ and } x \geq 0.$$ 

LP problem terminology

Slack variable: Variable added to problem to eliminate $\leq$ constraint.

Surplus variable: Variable added to problem to eliminate $\geq$ constraint.

Free variable: Unrestricted (read: not nonnegative) variable.

Convex set $C$: For $x, y \in C$ and $0 \leq \lambda \leq 1$, the linear combination $\lambda x + (1 - \lambda)y \in C$.

Extreme point $x \in C$: Is NOT a strict linear combination of two distinct points in $C$.

Let $S = \{x \mid Ax = b, x \geq 0\}$ be the feasible region of an LP problem.

LP feasible region properties

P0: The feasible region $S$ is a convex set.

P1: If $S \neq \emptyset$, then it has at least one extreme point.

P2: If a finite optima exists, then there is an optimal extreme point.

P3: If there are two distinct optimal (extreme) points, then there are infinitely many optimal solutions.

P4: The set of extreme points corresponds to the set of basic feasible solutions.