See Lecture 2 notes on recursively defined sequences.

**Recurrence relations**

A second-order linear recurrence relation with constant coefficients has the form

\[ a_n = ra_{n-1} + sa_{n-2} + f(n) \]

where \( r \) and \( s \) are constants and \( f(n) \) is some function of the integer \( n \). If \( f(n) = 0 \), the relation is **homogeneous**. Otherwise, it’s **nonhomogeneous**.

**Characteristic polynomial**

The recurrence relation \( a_n = ra_{n-1} + sa_{n-2} \) has a characteristic polynomial:

\[ p(x) = x^2 - rx - s. \]

The roots of \( p(x) \) are called the characteristic roots of \( a_n \).

**Theorem 1** Let \( x_1 \) and \( x_2 \) be the roots of the polynomial \( p(x) = x^2 - rx - s \), and let \( a_n = ra_{n-1} + sa_{n-2}, n \geq 2 \), be a homogeneous (!) second-order linear recurrence relation with constant coefficients. The solution to the recurrence relation \( a_n \) is

\[ a_n = \begin{cases} 
  c_1 x_1^n + c_2 x_2^n & \text{if } x_1 \neq x_2 \\
  c_1 x^n + c_2 n x^n & \text{if } x_1 = x_2 = x.
\end{cases} \]

The particular constants \( c_1 \) and \( c_2 \) are determined by the sequence’s initial conditions.

**How to solve HOMOGENEOUS S-O L RR w/CC**

**Step -1:** Check that the recurrence relation actually is second-order, linear, homogeneous, with constant coefficients.

**Step 0:** Find the characteristic polynomial \( p(x) \) for \( a_n \).

**Step 1:** Find the roots \( x_1, x_2 \) of \( p(x) \). Are they distinct?!!

**Step 2:** Depending on \( x_1 = x_2 \) or not, write down the solution with constants \( c_1, c_2 \).

**Step 3:** Solve for \( c_1 \) and \( c_2 \) using the initial conditions.

**Step 4:** Write down the complete solution for \( a_n \).