See Lecture 3 notes on solving S-O L RR w/CC that are HOMOGENEOUS.

**Recurrence relations**
Consider a nonhomogeneous S-O L RR w/CC with constants \( r, s \) and \( f(n) \neq 0; \)
\[
a_n = ra_{n-1} + sa_{n-2} + f(n)
\]

A particular solution of \( a_n \) is some function \( p_n \) such that \( p_n = rp_{n-1} + sp_{n-2} + f(n) \).
The associated homogeneous recurrence for \( a_n \) is \( a'_n = ra'_{n-1} + sa'_{n-2} \) which has a general solution \( q_n \) (constants \( c_1, c_2 \) still unspecified) using the characteristic polynomial method.

**Theorem 1** The sol'n to \( a_n = ra_{n-1} + sa_{n-2} + f(n) \), a nonhomogeneous S-O L RR w/CC, is
\[
a_n = p_n + q_n
\]
where \( p_n \) is a particular solution for \( a_n \) and \( q_n \) is the general solution for \( a'_n = ra'_{n-1} + sa'_{n-2} \).
The initial conditions for \( a_n \) determine the constants in the solution.

**How to solve NONhomogeneous S-O L RR w/CC**
**Step -1:** Check that all conditions (S-O, L, CC) are satisfied; \( a_n = ra_{n-1} + sa_{n-2} + f(n) \).
**Step 0:** GUESS the form of the particular solution based on \( f(n) \).
**Step 1:** Find a particular solution \( p_n \) to \( a_n \), ignoring initial conditions.
**Step 2:** Write down the associated homogeneous recurrence \( a'_n = ra'_{n-1} + sa'_{n-2} \).
**Step 3:** Find the (general) solution \( q_n \) to \( a'_n \), ignoring initial conditions.
**Step 4:** Write down the solution \( a_n = p_n + q_n \) with constants \( c_1, c_2 \).
**Step 5:** Solve for \( c_1 \) and \( c_2 \) using the initial conditions for \( a_n \).
**Step 6:** Write down the complete solution for \( a_n \).

**Generating functions**
The generating function for the sequence \( a_0, a_1, a_2, \ldots \) is the formal power series
\[
\sum_{i=0}^{\infty} a_i x^i.
\]
Generating functions can be added, multiplied, even differentiated. A couple of examples:
\[
\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i; \quad \frac{1}{1-ax} = \sum_{i=0}^{\infty} a^i x^i; \quad \frac{1}{(1-x)^2} = \sum_{i=0}^{\infty} (i+1)x^i
\]