Exam 1 is on **Wednesday, February 2nd**. It will cover Part I, Sec 5.1 – 5.4 and 8.2.

See Lecture 5 notes on algorithmic complexity.

**How to show** \( f = \mathcal{O}(g) \) **from the definition**

**Step 0:** Scratch work.

**Step 1:** Write down a **specific** integer \( n_0 \) (usually positive) and a **specific** real number \( c \).

**Step 2:** Demonstrate that \( |f(n)| \leq c|g(n)| \) for all \( n \geq n_0 \).

**How to show** \( f \neq \mathcal{O}(g) \) **from the definition**

**Step 0:** Scratch work.

**Step 1:** Write down an **arbitrary** integer \( n_0 \) (could be restricted to positive) and an **arbitrary** real number \( c \).

**Step 2:** Assume that \( |f(n)| \leq c|g(n)| \) for all \( n \geq n_0 \).

**Step 3:** Using this assumption, derived some contradiction.

**Step 4:** Conclude that \( f \) cannot be Big Oh of \( g \).

**How to show** \( f \preceq g \) **from the definition**

**Step 1:** Show \( f = \mathcal{O}(g) \) from the definition.

**Step 2:** Show \( g \neq \mathcal{O}(f) \) from the definition.

**How to show** \( f \asymp g \) **from the definition**

**Step 1:** Show \( f = \mathcal{O}(g) \) from the definition.

**Step 2:** Show \( g = \mathcal{O}(f) \) from the definition.

Alternatively . . .

**Theorem 1** Let \( f, g : \mathbb{N} \to \mathbb{R} \) be functions.

1. If \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \), then \( f \prec g \).

2. If \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \), then \( g \prec f \).

3. If \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = L \) for some \( L \in \mathbb{R}, L \neq 0 \), then \( f \asymp g \).