Solutions are to be written on the board.

1. Let \((G, \ast)\) be \(\mathbb{Z}_5^\ast\) under multiplication and \((H, \circ)\) be \(\mathbb{Z}_4\) under addition. Prove that these two groups are isomorphic. 
   
   *Note that \(\circ\) is a generic product symbol here (like \(\ast\)) and not function composition.*

2. If \(A\) and \(B\) are subgroups of a group \(G\), prove that \(A \cap B\) is a subgroup of \(G\).

3. Consider the dihedral group of order 8: \(D_8 = \langle a, x \mid a^2 = x^4 = e, axa^{-1} = x^{-1} \rangle\).

   (a) List all 10 subgroups of \(D_8\).
   
   (b) Which subgroups of \(D_8\) are isomorphic to the Klein 4-group? Justify your answer.

4. Consider the group of integers under addition. What is the cyclic subgroup generated by \(-1\)? What is \((2)\)? Are the odd integers a subgroup of \((\mathbb{Z}, +)\)? Justify your answers.

5. Let \(G\) be a group and \(a \in G\). Prove that the set \(A = \{a^i \mid i \in \mathbb{Z}\}\) is a subgroup of \(G\).

6. Prove or disprove from the definitions:

   (a) A cyclic group is abelian.
   
   (b) An abelian group is cyclic.