Solutions are to be written on the board.

1. Let $m > 1$ be an integer and $H_m = \{mx \mid x \in \mathbb{Z}\}$. Prove that $H_m$ is a subgroup of $(\mathbb{Z}, +)$.

2. Let $G$ be a group with identity $e$ and $H = \{a \in G \mid a^2 = e\}$.
   
   (a) Prove that if $G$ is abelian, then $H$ is a subgroup of $G$.
   
   (b) Is the statement true if $G$ is nonabelian? Justify your answer.

3. Let $(G, \ast)$ be a group and $H$ a nonempty subset of $G$ that is closed under $\ast$.
   
   (a) Prove that if $H$ is finite, then $H$ is a subgroup of $G$.
   
   (b) Is $H$ a subgroup of $G$ if $G$ is finite? Justify your answer.
   
   (c) What about if $G$ is infinite? Justify your answer.

4. Let $(G, \ast)$ and $(H, \circ)$ be two groups. Prove that if $\phi : G \to H$ is an isomorphism, then the inverse function $\phi^{-1}$ is an isomorphism from $H$ to $G$.

5. Let $G$, $H$, and $K$ be three groups. Prove that if $\phi : G \to H$ and $\psi : H \to K$ are isomorphisms, then the composition $\psi \circ \phi : G \to K$ is an isomorphism.

6. Let $\mathcal{G}$ be the set of all groups. Define a relation on $\mathcal{G}$ where $G \simeq G'$ if and only if $G$ is isomorphic to $G'$ for all $G, G' \in \mathcal{G}$. Prove that this is an equivalence relation.

7. Let $(G, \ast)$ be a group. Define a binary operation $\circ$ on $G$ by $a \circ b = b \ast a$.
   
   (a) Prove that $(G, \circ)$ is a group.
   
   (b) Prove that $(G, \ast)$ and $(G, \circ)$ are isomorphic.