1. Find a formula for the following summations. Prove your formula is correct using induction. *Hint: Write one proof outline and then edit as needed.*

(a) \[1 + 2 + 3 + \ldots + n\] for all integers \(n \geq 1\).
(b) \[1 + 2 + 2^2 + \ldots + 2^n\] for all nonnegative integers \(n\).

2. For which nonnegative integers \(n\) do the following inequalities hold? Prove your answer is correct using mathematical induction. *Hint: Write one proof outline and then edit as needed.*

(a) \[2n \leq n^2\]
(b) \[n^2 < 2^n\]
(c) \[2^n \leq n!\]
(d) \[n! < n^n\]

3. Prove that \(n^2 - 1\) is divisible by 8 whenever \(n\) is an odd positive integer.

4. Show that any postage that is a positive integer number of cents greater than 7 cents can be formed using just 3-cent stamps and 5-cent stamps.

5. (a) Determine which amounts of postage can be formed using just 5-cent and 6-cent stamps.
   (b) Prove your answer is correct using mathematical induction.
   (c) Prove your answer is correct using the “strong” form of mathematical induction.

6. Proof or spoof? Justify your answer.
   
   Claim: All horses are the same color. Proof: Let \(P(n)\) be the proposition that all horses in a set of \(n\) horses are the same color. Clearly, \(P(1)\) is true. Now assume that \(P(n)\) is true, so that all horses in any set of \(n\) horses are the same color. Consider any \(n + 1\) horses; number these as horses 1, 2, 3, \ldots, \(n\), \(n + 1\). Now the first \(n\) of these horses all must have the same color, and the last \(n\) of these must also have the same color. Since the set of the first \(n\) horses and the last \(n\) horses overlap, all \(n + 1\) must be the same color. This shows that \(P(n + 1)\) is true and finishes the proof by induction.