Math 6580

One must be sure that one has enabled science to make a great advance if one is to burden it with many new terms and require that readers follow research that offers them so much that is strange.

A. L. Cauchy

Linear Algebra, Infinite Dimensional Spaces, and MAPLE

PREFACE

These notes evolved in the process of teaching a beginning graduate course in Hilbert Spaces. The first edition was simply personal, handwritten notes prepared for lecture. A copy was typed and given to the students, for it seemed appropriate that they should have a statement of theorems, examples, and assignments. With each teaching of the course, the notes grew.

A text for the course has always been announced, but purchase of the text has been optional. A text provides additional reading, alternate perspectives, and a source of exercises. Whatever text was used, it was chosen with the intent of the course in mind.

This one quarter course was designed for science and engineering students. Often, as graduate students in the sciences and engineering mature, they discover that the literature they are reading makes references to function spaces, to notions of convergence, and to approximations in unexpected norms.

The problem they face is how to learn about these ideas without investing years of work which might carry them far from their science and engineering studies. This course is an attempt to provide a way to understand the ideas without the students already having the mathematical maturity that a good undergraduate analysis course could provide. An advantage for the instructor of this course is that the students understand that they need to know this subject.

The course does not develop the integration theory and notion of a measure that one should properly understand in order to discuss $L^2[0,1]$. Yet, these note suggest examples in that space. While this causes some students to feel uneasy with their unsophisticated background, most have enough intuition about integration that they understand the nature of the examples. The success of the course is indicated from the fact that science and engineering students often choose to come back the next term for a course in real variables and in functional analysis. The course has been provocative. Even those that do not continue with more graduate mathematics seem to feel it serves them well and provides an opening for future conversations in dynamics, control, and analysis.

In this most current revision of the notes, syntax for MAPLE has been added. Many sophomores leave the calculus thinking that the computer algebra systems are teaching tools because of the system's abilities to graph, to take derivatives, and to solve text-book differential equations. We hope, before they graduate, students will find that these systems really are a "way for doing mathematics." They provide a tool for arithmetic, for solving equations, for numerical simulations, for drawing graphs, and more. It's all in one program! These computer algebra systems will move up the curriculum.

In choosing MAPLE, I asked for a computer algebra system which is inexpensive for the students, which runs on a small platform, and which has an intuitive syntax. Also, MATLAB will run Maple syntax.

The syntax given in these notes is not always the most efficient one for writing the code. I believe that it has the advantage of being intuitive. One hopes the student will see the code and say, "I understand that. I can do it, too." Better yet, the student may say, "I can write better code!"

It is most important to remember that these notes are about linear operators on Hilbert Spaces. The notes, the syntax, and the presentation should not interfere with that subject.

The idea in these notes that has served the students best is the presentation of a paradigm for a linear operator on a Hilbert Space. That paradigm is rich enough to include all compact operators. For the student who is in the process of studying for various exams, it provides a system for thinking of a linear function that might have a particular property. For the student who continues in the study of graduate mathematics, it is a proper place to step off into a study of the spectral representations for linear operators.

To my colleagues, Neil Calkin and Eric Bussian, I say: The greatest complement you can give a writer is to read what he has written. To my students: These notes are better because they read them and gently suggested changes.

I am grateful to the students through the years who have provided suggestions, corrections, examples, and who have demanded answers for the assignments. In time.... In time....

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