Welcome to Math 4581:

Classical Mathematical Methods in Engineering

Course topics:
Fourier Series,
Recommended Readings for MATH 4581:

Electronic Text located at
http://www.mathphysics.com/pde/

Partial Differential Equations and
Boundary Value Problems with Maple

Introduction to Partial Differential Equations
with MATLAB

Boundary Value Problems

How to reach me: go to
www.math.gatech.edu/~herod/
Module 1: Linear Spaces

Definition: A **linear space of functions** is a collection of functions all having the same domain and

(a) if each of $f$ and $g$ belongs to the collection, then $f + g$ does also, and

(b) if $f$ is in the collection and $r$ is a number, then $r f$ is in the collection.
Examples: two of these collections are not linear spaces:

(1) \( C([0,1]) \).
(2) \( \mathbb{R}^3 \).
(3) points in \( \mathbb{R}^3 \) with last component zero.
(4) points in \( \mathbb{R}^3 \) with first component one.
(5) points \( \{x, y, z\} \) in \( \mathbb{R}^3 \) such that
\[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 4 & 5 \\
3 & 5 & 6
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}.
\]
(6) functions with period \( 2\pi \).
(7) positive functions on \([0,1]\).
(8) functions \( f \) in \( C([0,1]) \) with \( f(0) = f(1) = 0 \).
**Definition:** A basis for a vector space is a collection of linearly independent vectors \{ x_1, x_2, \ldots, x_n \} such that any vector in the space can be written as a linear combination of these.

**Examples:**

(1) \{ [1, -1, 0], [1, 1, 0], and [0, 0, 1] \} is a basis for \( \mathbb{R}^3 \).

(2) \{1, x, x^2, \ldots\} is a basis for the polynomials.

(3) \{ [1, -1, 1], [1, 1, 1], and [1, 0, 1] \} is not a basis for \( \mathbb{R}^3 \).
Definition: If $S$ is a collection of vectors, then the span of $S$ is all vectors that can be written as finite linear combinations of $S$.

Example:
The span of $\{ [1, -1, 1], [1, 1, 1], \text{ and } [1, 0, 1] \}$ is all vectors of the form $\{ [x, y, x] \}$.
Definition: A linear operator, say L, is a function with domain a vector space and for which

\[ L(a \cdot x + y) = a \cdot L(x) + L(y). \]

The nullspace, or kernel, for a linear operator is all vectors x such that \( L(x) = 0 \).

The range of a linear operator is all vectors y for which there is an x so that \( L(x) = y \).
We find the kernel and basis for the kernel using the linear differential operators

\[ L(y) = y'' - y' - 2y \]

and

\[ L(y) = y'' + y' + 2y. \]

Hint: \( r^2 - r - 2 = 0 \) has real roots

and

\[ r^2 + r + 2 = 0 \] has complex roots.
Home work: See Maple Worksheet for Module 1

Module 1 Summary:

A linear space of functions

A basis for a space

A span for a collection of vectors

A linear operator, its nullspace and range