Module 8: General Convergence

We discuss three types of convergence in C([0, 1]): normed, pointwise, and uniform.

Suppose we have a sequence of functions $f_1(x)$, $f_2(x)$, $f_3(x)$, ... converging to a function $g(x)$. We say that the $f$'s converge to $g$ in the sense of

**Norm Convergence** if

$$
\int_0^1 (f_n(x) - g(x))^2 \, dx \to 0 \text{ as } n \to \infty.
$$
Pointwise Convergence if, for each \( x \),

\[
  f_n(x) \to g(x) \text{ as } n \to \infty.
\]

Uniform Convergence if the maximum for all \( x \) in \([0, 1]\) of the difference in \( f_n(x) \) and \( g(x) \) goes to zero as \( n \to \infty \):

\[
  \max_{x \in [0,1]} |f_n(x) - g(x)| \to 0
\]

These methods of convergence can be contrasted.
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1. Uniform Convergence implies pointwise convergence. To see this, note only that if

\[
\max_{x \in [0,1]} |f_n(x) - g(x)| \to 0
\]

then for each \( x \), \( |f_n(x) - g(x)| \to 0 \).
These methods of convergence can be contrasted.

2. Uniform Convergence implies normed convergence. To see this, note that

\[
\int_0^1 (f_n(x) - g(x))^2 \, dx \leq \max_{x \in [0,1]} |f_n(x) - g(x)|^2
\]
3. Pointwise convergence does not imply uniform convergence. Each Max = 1/e
4. Pointwise convergence does not imply normed convergence.
5. Norm convergence does not imply pointwise convergence.
Assignment: See the Maple Worksheet

In this Module 8, we have
1. Discussed three general types of convergence in $C([0,1])$.
2. Contrasted these methods of convergence to determine which are the stronger, which are weaker.