Efficient modeling of spatially incoherent sources based on Wiener chaos expansion method for the analysis of photonic crystal spectrometers

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ABSTRACT

We present an efficient model for the simulation of spatially incoherent sources based on Wiener chaos expansion (WCE) method with two orders of magnitude shorter simulation time over the brute-force model. In this model the stochastic wave propagation equation is reduced to a set of deterministic partial differential equations (PDEs) for the expansion coefficients. We further numerically solve these deterministic PDEs by finite difference time domain (FDTD) technique. While the WCE method is general, we apply it to the analysis of photonic crystal spectrometers for diffuse source spectroscopy.

Keywords: Spatially incoherent source, Wiener chaos expansion, Photonic crystals.

1. INTRODUCTION

Photonic crystal (PC) structures[1, 2] since the introduction have been extensively investigated by many authors in all different fields of Optics. Two most cited properties of these artificial periodic structures are the existence of the photonic band gap[3] and the anomalous dispersion[4]. Where the first makes them a desired building block for the conventional integrated optic devices like waveguides[5] and cavities[6], the second makes them a super-dispersive element for the on-chip design of demultiplexers[7], PC beam splitters[8], and PC microspectrometers[9].

Nevertheless, in all of the applications proposed so far the source of incident light is perfectly coherent and spatially narrow. Our goal here is to develop a fast and efficient simulation tool to study the optical properties of photonic devices when the incoming light is spatially incoherent. This happens in many biomedical and environmental sensing applications in which the input light is diffuse. One example is Raman spectroscopy where the signal of interest is spatially incoherent[10]. In this paper, we present a new analytical model for a spatially incoherent source based on the recently developed Wiener chaos expansion (WCE) method[11]. We further implement this model using the finite difference time domain (FDTD) technique[12] and compare it to the brute-force model. We show by using WCE model, the simulation time is reduced considerably while the results are in close proximity to those produced by brute-force model.

In what follows, we introduce the WCE method in Section 2 and we further apply it to model the spatially incoherent source. The numerical implementation of the WCE model and the simulation results are presented in Section 3. In Section 4, the performance of this model in accuracy and simulation time is compared to the brute-force model. Concluding remarks are given in Section 5.

2. MODELING OF SPATIALLY INCOHERENT SOURCE BASED ON WIENER CHAOS EXPANSION METHOD

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In this section, we first introduce the spatially incoherent source as a stochastic signal in space and then we develop two different sets of formulation to model its propagation. The first formulation is nothing other than its abstract mathematical definition which is referred as the brute-force model and the second one is its modeling based on the lately proposed WCE method.

To simplify the mathematical formulation, the simulation domain has been reduced to a two-dimensional (2D) z-invariant medium. As shown in Figure 1, it is a 2D PC structure composed of square lattice of air holes in silicon. The radius of the air holes is $0.3a$, where a is the lattice constant. The one-dimensional source is placed in front of the PC along line A and the electric field intensity is monitored at the output line B. All the input sources are excited with a TE field (i.e., E_z, H_x, and H_y are nonzero).

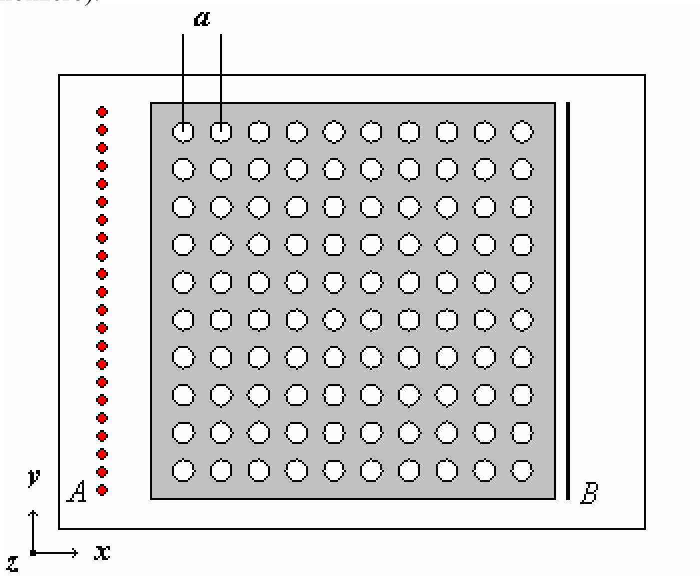

Figure 1. Schematic of a 2D square lattice PC structure composed of air holes in silicon. The radius of the air holes is $0.3a$ where a is the lattice constant. The size of the structure is $10a$ by $10a$. The spatially incoherent source is along line A and the output electric field intensity is calculated at line B.

The electromagnetic wave propagation throughout any structure is governed by the Helmholtz wave equation,

$$\nabla^2 E_z(x,y,t) - \mu\varepsilon(x,y)\frac{\partial^2 E_z(x,y,t)}{\partial t^2} = \mu\frac{\partial J_z(x,y,t)}{\partial t} \tag{1}$$

where the current density (J_z) is the source of excitation. For the modeling of spatially incoherent source any two point sources on line A should radiate independently (or incoherently) of the other. This definition by itself is the immediate technique for numerical modeling of a spatially incoherent source and it has been addressed as the brute-force model in Reference [13]. In this analysis, the output response to each point source along line A is calculated independently, and the results are added incoherently (i.e., in intensity)[14]. The main drawback of this technique is that the number of simulations of the entire structure is equal to the number of the point sources along line A resulting in very long overall simulation times. Therefore, we only use the brute-force technique as a reference for the comparison of the simulation time and accuracy of the other methods. Any efficient technique should result in simultaneous simulation of the effect of different point sources along line A and impose the incoherent nature by adding statistical randomness to the sources themselves.

As it is clear from the title of spatially incoherent source, the origin of randomness is in the spatial function of the source and it is deterministic in the time domain. To improve the simulation time, we propose to use white noise (i.e., the derivative of Brownian motion) to model the spatial part of the current density as well as electric field (E_z) by random signals in space (along y here). More precisely, we model the source $J_z(x_A, y, t)$ along line A as

$$J_z(y,t) = dW(y)V(t) \tag{2}$$

where $V(t)$ is a deterministic function and $W(y)$ is a Brownian motion. This random source makes the Equation (1) a stochastic PDE. By choosing any orthonormal basis $m_i(y)$, we can introduce independent Gaussian random variables such that

$$dW(y) = \sum_i \xi_i m_i(y) \tag{3}$$

with

$$\xi_i = \int_0^y m_i(s) dW(s) \quad i = 1, 2, \ldots . \tag{4}$$

Similarly, we can decompose the electric field and the current density on these orthonormal bases and plug them back into Equation (1). Therefore, the stochastic PDE is reduced to a set of deterministic PDEs for the expansion coefficients. It has been proved that all the statistical moments of the random solution of the original PDE at the output line *B* can be directly calculated using these expansion coefficients[11]. Obviously depending on the number of expansion terms considered in Equation (3), the accuracy and the gain in simulation time is limited. However, as shown in Reference [11] WCE is a very fast converging expansion technique. Thus, by truncating the Equation (3) to a small number of terms, we can achieve accurate enough results in a very fast simulation for almost all practical PC structures.

3. SIMULATION RESULTS

The structure under simulation is a 2D PC as shown in Figure 1. For computation facility we have limited the size of the PC to a 10*a* by 10*a* square lattice. We implement both spatially incoherent source models in time domain using the standard FDTD method. The *x*-*y* plane is discretized so that we get 24 grid cells per lattice constant along *x* and *y* axes. The source line *A* is placed one lattice constant (i.e., 24 grid cells) before the PC structure and the output line *B* is fixed 3 grid cells after PC structure. To imitate an infinite environment a 12 grid cell perfectly matched layer (PML) is set up around the structure to remove the non-physical reflections from the boundaries.

In brute-force simulation, we have assigned one point source per each grid point along line *A*. Since the length of the input source line is 10*a*, it results to 240 point sources along it. In each individual simulation, only one source is excited with a sinusoidal modulated Gaussian pulse as

$$V(t) = \sin\big(\omega(t - t_0)\big)\exp\left(-\left(\frac{t - t_0}{T}\right)^2\right) \tag{5}$$

and it propagates all the way through the structure to get to the output line *B*. The center frequency of the pulse (ω/c) is 0.04 and its width ($\Delta\omega$) is 0.016. The width of the Gaussian pulse (T) in time domain is $1/\Delta\omega = 62.5$ which corresponds to 120 time steps in our FDTD simulation. As previously stated in Ref. [13], each simulation takes 2^16 time steps to end up with the steady state result at the output line *B*. This sums up to the total simulation time of 240 * 2^16 time steps for the brute-force model.

However, in WCE model the number of simulations is as large as the number of expansion coefficients. The smaller number of expansion coefficients results to the faster overall simulation time. Each expansion coefficient of the current density is a product of a time function $V(t)$ and a space function $m_i(y)$. For the numerical simulation here, we have chosen a sinusoidal basis function for $m_i(y)$,

$$m_0(y) = \frac{1}{\sqrt{y_f - y_0}}$$

$$m_i(y) = \frac{1}{\sqrt{2(y_f - y_0)}} \cos((i-1)\pi \frac{y - y_0}{y_f - y_0}) \tag{6}$$

where y_f is the total length of the input line *A* which is 240 grid cells. It is worth mentioning that in general we can choose any orthonormal basis for the spatial function.

Due to the fast convergence behavior of the WCE method we can reduce the simulation time considerably. Figure 2 shows the simulation result of electric field intensity as a function of the normalized frequency at a typical point on the output line *B* in Figure 1. The red solid curve is the result of the brute-force model and the blue dashed curve is the result of our WCE-based technique with only 10 expansion coefficients. So instead of repeating one simulation for each individual point source along line *A*, we do one simulation for each spatial orthonormal basis function used along line *A*. For this result we have used the first 10 harmonics of sinusoidal function for the orthonormal basis functions. Similar to the brute-force simulation each individual simulation takes 2^16 time steps. The agreement of the two techniques is clear from Figure 2 with our proposed WCE-based technique having 24 times shorter computation time.

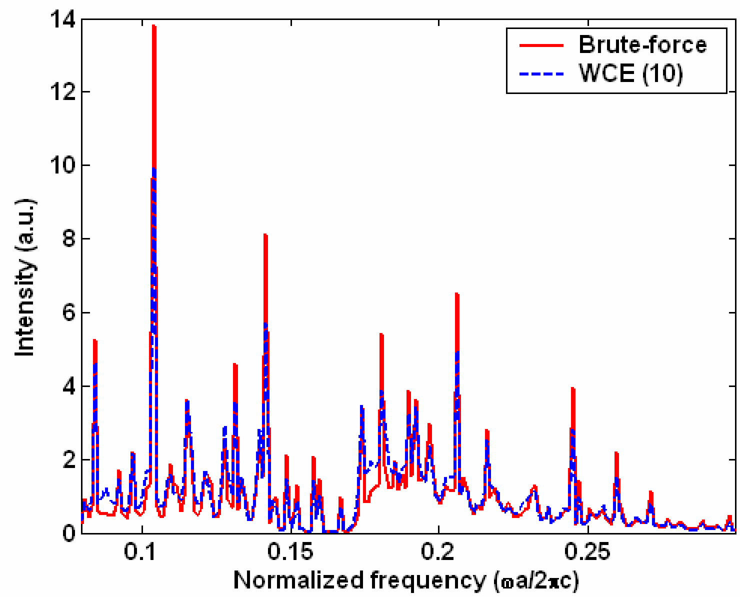

Fig. 2. The electric field intensity as a function of normalized frequency at a single point on line *B* in Figure 1 calculated using the brute-force model and our proposed WCE-based model with only 10 expansion coefficients. Good agreement between the two models is clear.

4. COMPARISON OF THE WCE MODEL WITH THE BRUTE-FORCE MODEL

The main trade-off in comparison of the two models is the relative error versus the gain in simulation time. The relative error is calculated by the total power of the differences between the intensity spectra of the two models for all points along line *B* divided by the total power of the intensity spectrum of the brute-force model. As seen in Figure 3(a), by increasing the number of expansion coefficients for the WCE model we get smaller and smaller percentage error with respect to the brute-force simulation. In other words it more accurately resembles the exact result. However, increasing the number of expansion coefficients obviously degrades the gain in simulation time as illustrated in Figure 3(b). As seen from Figure 3 by accepting 4% error which is absolutely fine for the most of practical applications, the simulation of the WCE model is finished 24 times faster than the simulation of the brute-force model.

The efficiency of the WCE model is more clearly declared for the larger size PC structures. For the second simulation we have used a twice long PC structure with the same specifications as the previous one. Here the size of the structure is 20*a* by 10*a* as shown in Figure 4. Assuming 24 point sources per each lattice constant, it adds up to 480 point sources along the input line *A*. Since the width of the structure is fixed (i.e., 10*a*), as before the simulation time of each point source takes 2^16 time steps to end up with the steady state result at the output line *B*. As a result the overall simulation time of the brute-force model for this structure is 480 * 2^16 time steps.

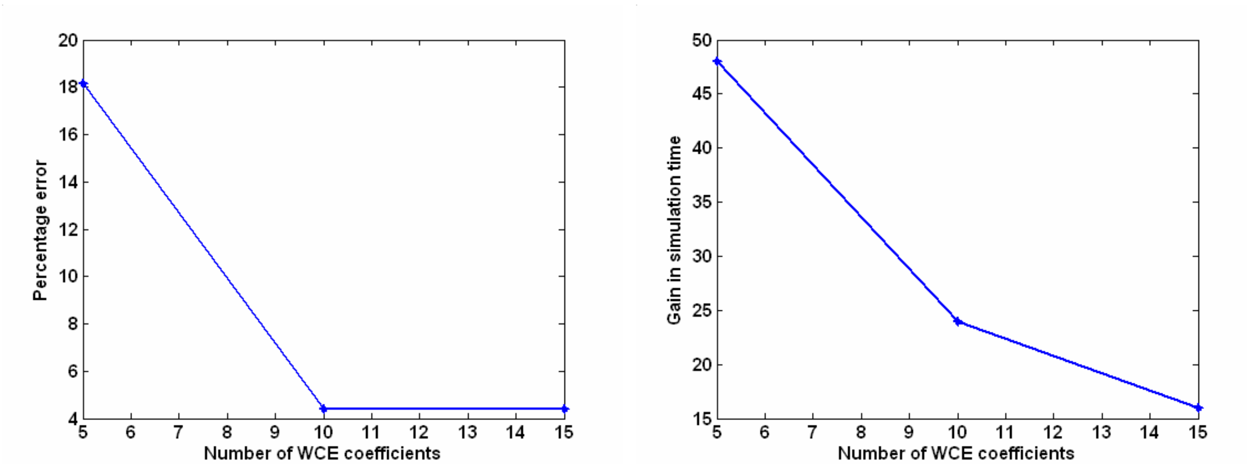

Figure 3. (a) The percentage relative error of the WCE model with respect to the brute-force model versus the number of expansion coefficients (b) The gain in simulation time of the WCE model compared to the brute-force model versus the number of expansion coefficients.

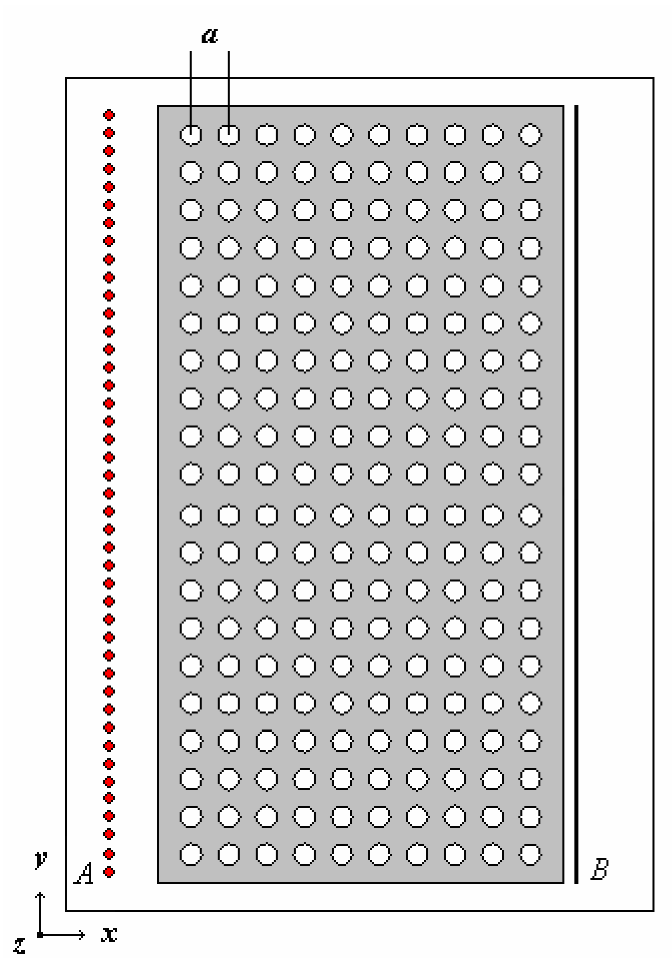

Figure 4. Schematic of a 2D square lattice PC structure composed of air holes in silicon. The radius of the air holes is 0.3*a* where *a* is the lattice constant. The size of the structure is 20*a* by 10*a*. The spatially incoherent source is along line *A* and the output electric field intensity is calculated at line *B*.

However, by the simulation of WCE model with only 15 expansion coefficients we can get almost exact result as the brute force model. Once again Figure 5 shows the simulation result of electric field intensity as a function of the normalized frequency at a typical point on the output line *B* in Figure 4. As obvious from Figure 5, the blue dashed curve of WCE model is almost in perfect match with the red solid curve of the brute-force model. Hence, the WCE model simulates the spatially incoherent source 32 (i.e., 480/15) times faster than the brute-force model does with the same accuracy.

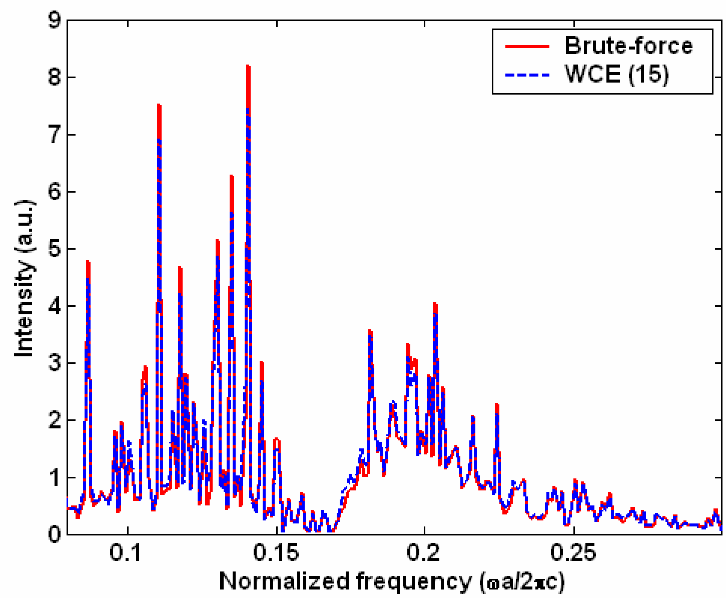

Figure 5. The electric field intensity as a function of normalized frequency at a single point on line *B* in Figure 4 calculated using the brute-force model and our proposed WCE-based model with only 15 expansion coefficients. There is almost a perfect match between the two models.

What this second simulation suggests is that the efficiency of the WCE model significantly improves in both accuracy and simulation time for larger structures. Moreover, the number of expansion coefficients for the fixed amount of relative error is not doubled for the twice long structure in the recent simulation. For instance as seen from Figure 6, for getting close to the 4% error as the first simulation we have to roughly consider 13 expansion coefficients which is 3 coefficients more than what we have considered for the smaller size structure and not twice of that. Consequently, for a practical size PC structures which are almost 80*a* long, we expect about 20 expansion coefficient to get to the vicinity of the 4% error. On the other side for the simulation of the brute-force model we need to run 80 * 24 = 1920 individual simulations. Thus, compared to the brute-force model, the WCE model simulates the spatially incoherent source with two orders of magnitude shorter (i.e., 1920/20 = 96) computation time.

As the last comment in this section, we will show that the brute-force model by itself is a special case of the WCE model where the spatial orthonormal basis functions are the Dirac δ functions. Assume that we have two sets of spatial basis functions that can be used for WCE model. One of them is δ-basis functions, $n_i(y)$, and the other is the sinusoidal basis functions, $m_i(y)$, which we have used so far. By rewriting the Equation (3) for these two orthonormal bases we get

$$dW(y) = \sum_i \xi_i m_i(y) = \sum_i \eta_i n_i(y) . \tag{7}$$

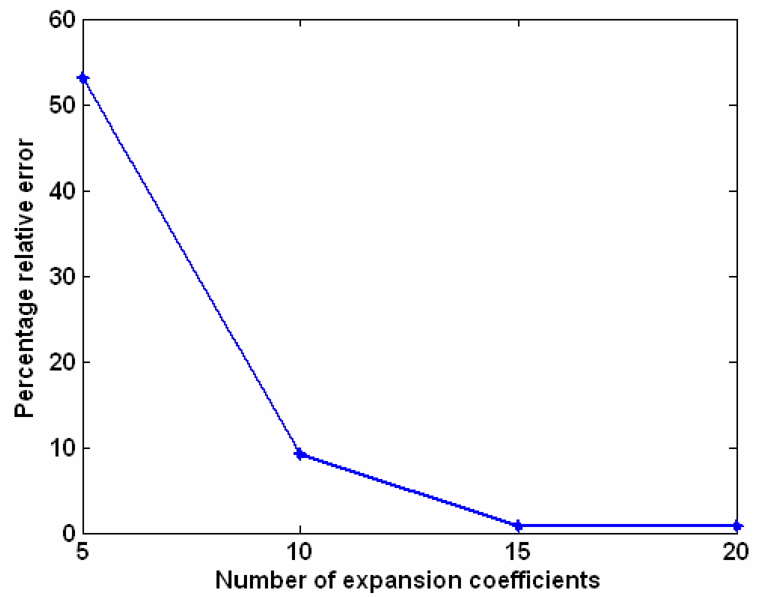

Figure 6. The gain in simulation time of the WCE model compared to the brute-force model versus the number of expansion coefficients for the PC structure shown in Figure 4. With only 15 expansion coefficients we get almost zero error.

For the brute-force model the source term in the right hand side of the Equation (1) is expanded using δ-basis functions

$$J_{zBF}(y,t) = \sum_i \eta_i n_i(y) V(t) = \sum_i \eta_i \delta(y - y_i) V(t) . \tag{8}$$

Now we can rewrite this equation by using Equation (7) as following

$$J_{zBF}(y,t) = \sum_i \eta_i \delta(y - y_i) V(t) = \sum_i \xi_i m_i(y) V(t) = J_{zWCE}(y,t) . \tag{9}$$

Therefore, the brute-force model is a WCE model with the spatial δ-basis functions. Since the brute-force model is the most time-costly model, the δ-basis functions are the least optimum choice for the orthonormal basis functions in the WCE model.

5. CONCLUSIONS

We have developed a new model based on WCE technique to simulate the spatially incoherent source for the analysis and optimization of the PC devices under diffuse light illumination. We showed that the efficiency of the WCE model is well improved in both accuracy and simulation time for larger size PC structures. We believe that for practical size PC devices up to two orders of magnitude faster simulation with almost perfect accuracy is achievable. We further proved that the brute-force model is the least optimum WCE model for the simulation of spatially incoherent source. In other words, choosing any orthonormal basis function other than Dirac δ function reduces the simulation time.

Although we have simulated this model in a 2D space, it can be easily generalized for simulation of a spatially incoherent source in a 3D space where the input source radiates from a surface. Moreover, the WCE technique can be applied to many other PDEs, linear or nonlinear, with the spatial or temporal stochastic input source.

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