MATH 4280 – Hurley – homework problems April 14, 2008

1. Let $(\Gamma, \mathcal{A}, \mathcal{B})$ be an information channel with capacity C, and suppose that $H(\mathcal{A}|\mathcal{B}) = 0$. Prove that for all codes $\mathcal{C} \subseteq A^n$ with rate R, $R \leq C$.

2. Let Γ be a binary symmetric channel. Consider the following channels each with input and output alphabets $\mathbb{F}_2^3$, the set of binary words of length 3. Let $a_1a_2a_3$ be the input and $b_1b_2b_3$ be the output.

   (a) The output b_1 is determined by sending a_1 through the binary symmetric channel Γ. The outputs b_2 and b_3 are chosen randomly with each possibility being equally likely.

   (b) Output b_1 is determined by sending the majority bit from $a_1a_2a_3$ through the binary symmetric channel Γ. Then determine b_2 and b_3 by letting $b_3 = b_2 = b_1$.

   (c) Output b_k is determined by sending a_k through the binary symmetric channel Γ

   For each channel, assume a uniform input distribution, and a decision rule that simply declares that the input is the same as the output. Find the probability of a decision error in each case.