

MATH 4280 – Hurley – homework problem February 29, 2008

1. Let  $\Gamma$  be an information channel with input source  $\mathcal{A}$ , output source  $\mathcal{B}$  and channel matrix  $\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 2/3 & 1/3 \end{pmatrix}$ . Suppose  $\mathcal{A}$  has alphabet  $A = \{0, 1\}$  and probabilities  $p_0 = \frac{3}{4}$  and  $p_1 = \frac{1}{4}$ , and  $\mathcal{B}$  has alphabet  $B = \{0, 1, 2\}$  and probabilities  $q_0, q_1, q_2$ . In class we determined that the probabilities for  $\mathcal{B}$  are  $q_0 = \frac{3}{8}$ ,  $q_1 = \frac{17}{48}$  and  $q_2 = \frac{13}{48}$ , and that the backward probabilities  $Q_{ij}$  are  $Q_{00} = 1$ ,  $Q_{10} = 0$ ,  $Q_{01} = \frac{9}{17}$ ,  $Q_{11} = \frac{8}{17}$ ,  $Q_{02} = \frac{9}{13}$  and  $Q_{12} = \frac{4}{13}$ .

Find the following entropies using  $\log_2$ .

- The input entropy and the output entropy.
- The conditional entropies  $H(\mathcal{A}|0)$ ,  $H(\mathcal{A}|1)$ ,  $H(\mathcal{A}|2)$  and the equivocation of  $\mathcal{A}$  with respect to  $\mathcal{B}$ .
- The conditional entropies  $H(\mathcal{B}|0)$ ,  $H(\mathcal{B}|1)$ , and the equivocation of  $\mathcal{B}$  with respect to  $\mathcal{A}$ .
- The joint entropy  $H(\mathcal{A}, \mathcal{B})$ .
- Check that your answers satisfy equations 4.6 and 4.7 on page 63 of the textbook.