

MATH 4280 – Hurley – homework problem March 10, 2008

1. Let Γ be a binary symmetric channel with $P = \frac{9}{10}$ and $p = \frac{3}{4}$.
 - (a) Find $H(\mathcal{A}|\mathcal{B})$ using base 2.
 - (b) For 0 and 1 in the output source alphabet B , describe optimal binary codes C_0 and C_1 for \mathcal{A} given $b = 0$ and $b = 1$ respectively, find their average word lengths, and find L_1 , the average word length of the encoding of \mathcal{A} given by using C_0 when $b = 0$ and C_1 when $b = 1$.
 - (c) Now use Γ^2 and for each $\mathbf{b} \in B^2$ describe an optimal binary code $C_{\mathbf{b}}$ for \mathcal{A}^2 , find the average word length of each $C_{\mathbf{b}}$, and find L_2 the average word length of the encoding of \mathcal{A}^2 given by using $C_{\mathbf{b}}$ when \mathbf{b} is received.
 - (d) Check that your answers satisfy

$$H(\mathcal{A}|\mathcal{B}) \leq \frac{L_n}{n} < H(\mathcal{A}|\mathcal{B}) + \frac{1}{2},$$

from page 69 of the text.