MATH 4280 – Hurley – homework problem February 1, 2008

1. Find an r-ary Huffman encoding for a source with the probabilities $p_i = 0.3, 0.2, 0.2, 0.1, 0.1, 0.1$ for $r = 3$ and $r = 4$

2. Find an r-ary Huffman encoding for a source with the probabilities $p_i = 0.8, 0.11, 0.05, 0.02, 0.01, 0.005, 0.005$ for $r = 3$, $r = 4$ and $r = 5$.

1. $q = 6$, $r = 3$, Find $2 \leq s \leq 3$ such that $6 \equiv s \pmod 2$

$6 \equiv 2 \pmod 2$, so first reduction is by 2

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$

.3 .2 .2 .1 .1 .1

$s_7 = s_5 \vee s_6 \quad p_7 = .2$

$S' = \{s_1, s_2, s_3, s_7, s_4\}$

.3 .2 .2 .2 .1

$s_8 = s_3 \vee s_7 \vee s_4 \quad p_8 = .5$

$S'' = \{s_8, s_1, s_2\}$

.5 .3 .2

$s_9 = s_8 \vee s_1 \vee s_2 \quad p_9 = 1$

$S''' = \{s_9\}$

$s_9 \longrightarrow \varepsilon$

$s_8 \longrightarrow 0$

$s_1 \longrightarrow 1$

$s_2 \longrightarrow 2$

$s_3 \longrightarrow 00$

$s_7 \longrightarrow 01$

$s_4 \longrightarrow 02$

$s_5 \longrightarrow 010$

$s_6 \longrightarrow 011$

$C = \{1, 2, 00, 02, 010, 011\}$

$L(C) = 1 + .5 + .2$

$= 1.7$

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Alternate $q=6,\ r=3$

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$
$.3\quad .2\quad .2\quad .1\quad .1\quad .1$

$s_7 = s_5 \vee s_6 \quad p_7 = .2$

$S' = \{s_1, s_7, s_2, s_3, s_4\}$
$.3\quad .2\quad .2\quad .2\quad .1$

$s_8 = s_2 \vee s_3 \vee s_4 \quad p_8 = .5$

$S'' = \{s_8, s_1, s_7\}$
$.5\quad .3\quad .2$

$s_9 = s_8 \vee s_1 \vee s_7 \quad p_9 = 1$

$S''' = \{s_9\}$

$s_9 \longrightarrow \varepsilon$
$s_8 \longrightarrow 0$
$s_1 \longrightarrow 1$
$s_7 \longrightarrow 2$
$s_2 \longrightarrow 00$
$s_3 \longrightarrow 01$
$s_4 \longrightarrow 02$
$s_5 \longrightarrow 20$
$s_6 \longrightarrow 21$

$C = \{1, 00, 01, 02, 20, 21\}$

1) $q=6\quad r=4$, Find $2 \le s \le 4$ s.t. $6 \equiv s \pmod 3$

$6 \equiv 3 \pmod 3$, first reduction is by 3

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$
$.3\quad .2\quad .2\quad .1\quad .1\quad .1$

$s_7 = s_4 \vee s_5 \vee s_6 \quad p_7 = .3$

$S' = \{s_7, s_1, s_2, s_3\}$
$.3\quad .3\quad .2\quad .2$

$s_8 = s_7 \vee s_1 \vee s_2 \vee s_3 \quad p_8 = 1$

$S'' = \{s_8\}$

$s_8 \longrightarrow \varepsilon$
$s_7 \longrightarrow 0$
$s_1 \longrightarrow 1$
$s_2 \longrightarrow 2$
$s_3 \longrightarrow 3$
$s_4 \longrightarrow 00$
$s_5 \longrightarrow 01$
$s \longrightarrow 02$

$C = \{1, 2, 3, 00, 01, 02\}$

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2. Find an r-ary Huffman encoding for a source with the probabilities $p_i = .8, .11, .05, .02, .01, .005, .005$, for $r=3$, $r=4$ and $r=5$

$q = 7$, $r = 3$, $7 \equiv 3 \pmod 2$, so $s = 3$.

$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$
$\quad .8 \quad .11 \quad .05 \quad .02 \quad .01 \quad .005 \quad .005$

$s_8 = s_5 \vee s_6 \vee s_7 \quad p_8 = .02$

$S' = \{s_1, s_2, s_3, s_4, s_8\}$
$\quad .8 \quad .11 \quad .05 \quad .02 \quad .02$

$s_9 = s_3 \vee s_4 \vee s_8 \quad p_9 = .09$

$S'' = \{s_1, s_2, s_9\}$
$\quad .8 \quad .11 \quad .09$

$s_{10} = s_1 \vee s_2 \vee s_9 \quad p_{10} = 1.$

$s_{10} \longrightarrow \varepsilon$
$s_1 \longrightarrow 0$
$s_2 \longrightarrow 1$
$s_9 \longrightarrow 2$
$s_3 \longrightarrow 20$
$s_4 \longrightarrow 21$
$s_8 \longrightarrow 22$
$s_5 \longrightarrow 220$
$s_6 \longrightarrow 221$
$s_7 \longrightarrow 222$

$C = \{0, 1, 20, 21, 220, 221, 222\}$

$q = 7$, $r = 4$, $7 \equiv 4 \pmod 3$, so $s = 4$

$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$
$\quad .8 \quad .11 \quad .05 \quad .02 \quad .01 \quad .005 \quad .005$

$s_8 = s_4 \vee s_5 \vee s_6 \vee s_7, \; p_8 = .04.$

$S' = \{s_1, s_2, s_3, s_8\}$
$\quad .8 \quad .11 \quad .05 \quad .04$

$s_9 = s_1 \vee s_2 \vee s_3 \vee s_8, \; p_9 = 1.$

$S'' = \{s_9\}$

$s_9 \longrightarrow \varepsilon$
$s_1 \longrightarrow 0$
$s_2 \longrightarrow 1$
$s_3 \longrightarrow 2$
$s_8 \longrightarrow 3$
$s_4 \longrightarrow 30$
$s_5 \longrightarrow 31$
$s_6 \longrightarrow 32$
$s_7 \longrightarrow 33$

$C = \{0, 1, 2, 30, 31, 32, 33\}$

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2) $r = 5$

$q = 7$, $r = 5$, $7 \equiv 3 \pmod 4$

$S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$
.8 .11 .05 .02 .01 .005 .005

$S_8 = S_5 \vee S_6 \vee S_7$ $p_8 = .02$

$S' = \{S_1, S_2, S_3, S_4, S_8\}$
.8 .11 .05 .02 .02

$S_9 = S_1 \vee S_2 \vee S_3 \vee S_4 \vee S_5$ $p_9 = 1$

$S'' = \{S_9\}$

$S_9 \longrightarrow \varepsilon$
$S_1 \longrightarrow 0$
$S_2 \longrightarrow 1$
$S_3 \longrightarrow 2$
$S_4 \longrightarrow 3$
$S_8 \longrightarrow 4$
$S_5 \longrightarrow 40$
$S_6 \longrightarrow 41$
$S_7 \longrightarrow 42$

$C = \{0, 1, 2, 3, 40, 41, 42\}$